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Optical Properties of Lithium Fluoride in the Extreme Ultraviolet[1]

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The absorption coefficient of lithium fluoride in the extreme ultraviolet was measured for 50 crystals produced in the laboratory. The variation in the absorption coefficients for the different crystals was almost as great as that found in natural fluorite although the individual specimens seemed more homogeneous than natural crystals. Numerical values of the absorption coefficient are given for the best specimen. The light lost from the transmitted beam on passing through a surface was studied for cleavage and polished surfaces. At short wave-lengths the loss of light was much greater than that expected for specular reflection. Tests with various cleaning agents indicated that at least a part of this loss of light was due to a layer of foreign material adhering to the surface. Considerable gain in transmission especially at short wave-lengths was obtained by careful cleaning of the polished surfaces with a highly volatile solvent followed by heating to a temperature just below red heat. The index of refraction curve for lithium fluoride was extended into the Schumann region by a modified minimum deviation method.

LARGE lithium fluoride crystals, more transparent in the extreme ultraviolet than fluorite, can be produced in the laboratory. These crystals can be cut and polished with about the same ease as fluorite and may be used as a substitute for fluorite in many optical instruments for use in the Schumann region.

PREPARATION OF THE CRYSTALS

The crystals were made by the method of Bridgman[2] which consists of mechanically lowering a crucible containing the molten salt through the core of an electric furnace. Although a vacuum furnace is not necessary, it is desirable because some constituent of the air, probably water vapor, slightly attacks the melted salt. The furnace was built with a sufficiently large region of uniform high temperature to melt all of the salt at one time. Below this region there was a high temperature gradient down to room temperature. This gradient prevents the formation of a polycrystalline mass at the top of the crucible. The time required for the formation and annealing of a crystal about three centimeters across was approximately thirty-six hours.

The crucibles made from platinum foil were cleaned by boiling in nitric acid, washing with distilled water and heating in a gas-air flame until no color was imparted to the flame by salts remaining on the surface.

Commercial lithium fluoride is unsuitable for the production of crystals transparent to the extreme ultraviolet since it contains a considerable amount of impurity. The material which gave the best result was prepared by mixing hydrofluoric acid, purified by distillation, with lithium nitrate which had been recrystallized several times. Platinum vessels were used for all steps of the purification.

[1] A preliminary report of these experiments was presented at the Washington Meeting, April 25, 1935.
[2] Bridgman, Proc. Am. Acad. **60**, 305 (1925).

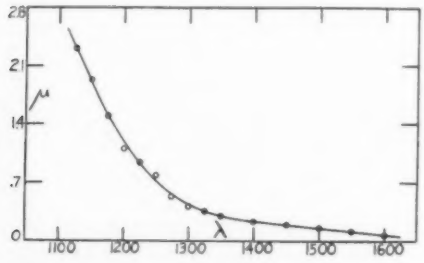

FIG. 1. Absorption coefficient per centimeter.

TRANSMISSION

The transmission of the lithium fluoride was measured photographically by the apparatus and methods described in a previous paper.[3] It is customary to represent the transmission, T, of a transparent plate of material by the expression

$$T=(1-r)^2e^{-\mu d},$$

where r is the reflection coefficient at normal incidence, μ the absorption coefficient, and d the thickness. It is immediately obvious that the reflection and absorption coefficients may be determined from the transmissions of two thicknesses of the material providing r and μ are the same for the two pieces.

ABSORPTION COEFFICIENT

In order to obtain a representative value for the absorption coefficient, it is necessary to take a number of sections from different portions of the crystal. A series of measurements on five plates cut from the same crystal and having a range of thickness from 25.0 mm to 0.5 mm showed that the absorption coefficient did not vary by more than ten percent throughout the crystal. The results for three pieces of another crystal showed about the same variation although the absolute value of the absorption coefficient was different. These measurements indicate that the lithium fluoride crystals were slightly more homogeneous than the fluorite crystals examined by W. M. Powell.[4]

Although the absorption coefficient was reasonably constant throughout a single crystal, there was an extreme variation between crystals. For example, crystals grown from commercial lithium fluoride were completely opaque below 1600A, while specimens of the same thickness (1 to 3 mm) made from the purified salt transmitted between 80 and 90 percent of the light in this region. Crystals grown from the same batch of salt under conditions as nearly identical as possible may have quite different absorption coefficients. This variation is probably due to a trace of impurity although spectroscopic analysis failed to show any definite differences in chemical constitution between the most transparent and the most opaque specimens.

Fig. 1 gives the absorption coefficient as a function of wave-length for the best of fifty lithium fluoride crystals tested. In most cases a qualitative test of the transmission near 1100A was sufficient to show that the absorption coefficient of a crystal was much higher than that of the best specimen.

LOSS OF LIGHT AT THE SURFACE

For an ideal surface the reflection coefficient, r, in the above formula is the fraction of light specularly reflected by a single surface. However, r will usually include other factors since any surface mechanism which removes light from the transmitted beam will be included in this term of the equation. Hence it is desirable to define r as the total fractional loss of light at a surface rather than as the reflection coefficient. Factors which reduce the intensity of a beam of light passing through the surface are, in addition to regular reflection, diffusion due to roughness, and absorption in a surface layer.

When the surface loss of light is computed from the transmissions of two or more thicknesses of a substance, it is necessary to have all of the surfaces polished in the same manner. A test of the reproducibility of the surfaces was made by repolishing one piece of lithium fluoride five times. Care was taken to repeat the conditions of polishing with as little variation as possible. These measurements showed that the transmission for the piece did not change by more than three percent. If repolishing changes the transmission of a plate of material, it must change the surface loss because a decrease in thickness by a few thousandths of a millimeter

[3] Schneider, Phys. Rev. **45**, 152 (1934).
[4] Powell, Phys. Rev. **45**, 154 (1934).

should not alter the absorption in the body of the crystal. Hence the loss of light at the two surfaces did not vary by more than three percent of the total transmission on repolishing. Furthermore, since the limit of accuracy of the photographic method of measuring intensity of light is of the order of magnitude of three percent, the surfaces may be considered reproducible to within experimental error. Curves *C*, *D*, *E*, and *F* in Fig. 2 represent different surface conditions due to differences in the method of polishing for the same pieces of lithium fluoride. Since the same pieces of lithium fluoride were used in obtaining these four curves, the relative positions of the curves are accurate to about ± 0.015 in the fractional loss of light at the surface. It should be noted, however, that an error in the absorption coefficient due to the inhomogeneities in the crystals might cause a consistent error in the absolute values of the surface loss of as much as 20 to 30 percent. Such an error would displace the curves relative to the axis but would shift all of them by about the same amount.

Curve *A* represents the fraction of light specularly reflected by a perfect surface of lithium fluoride. The values for this curve were calculated by Fresnel's equation from the index of refraction alone, the extinction coefficient being too small in this region to alter the result. The divergence between the theoretical reflecting power curve and the experimental curves shows that a perfect surface was never obtained.

The surfaces used in measuring curve *B* were prepared by cleaving the crystals. All of these surfaces were somewhat irregular and it is quite possible that the waviness accounts for the entire discrepancy between this and the theoretical curve. That portion of a wavy surface which is not normal to the incident beam should reflect a greater fraction of the light since the reflection increases with the angle of incidence. Furthermore, such a surface should refract an appreciable part of the light in directions other than that of the transmitted beam. Since both of these factors become more important as the index of refraction becomes higher, the increasing divergence between the theoretical and the experimental curves is reasonable.

The other curves in Fig. 2 give the fractional loss of light for a polished surface. In all cases

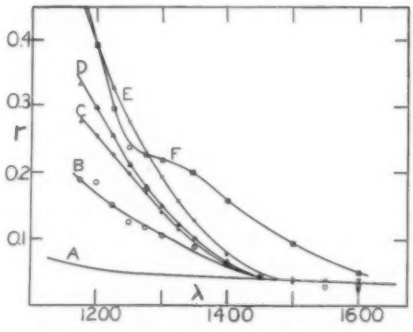

FIG. 2. Fractional loss of light at a single surface. Curve *A*, theoretical reflecting power; *B*, cleavage surface; *C*, *D*, *E*, and *F* polished surfaces cleaned, *C* by heating, *D* with ethyl ether, *E* with petroleum ether, *F* with xylene.

the polishing was done with rouge on silk lubricated with water, but the cleaning following the polishing was varied. Felt and lead were tried as polishing laps, but both scratched the surface badly. Although a beeswax lap with rouge gave as good a polish as the silk, it proved unsatisfactory because the layer of wax remaining on the surface could not be removed completely. Commercial xylene was used to clean the surface shown in curve *F*. Although this liquid was almost certain to leave a layer of practically nonvolatile material on the surface, it was tried because it is frequently used alone or with oil as a polishing lubricant. Curves *D* and *E* are for surfaces cleaned with medical ethyl ether and with petroleum ether. These agents were chosen as substances which were not apt to leave a residue on the surface. Acids and bases cannot be used for cleaning because they etch the surface.

The differences between the curves for the polished surfaces may be explained by the assumption that a layer of material other than lithium fluoride is present on the surface. The flat portion of the xylene curve near 1300A is particularly hard to explain on any other hypothesis. This departure from the trend of the curve is about twice as great as the experimental error. The gradually increasing separation with decreasing wave-length between the other curves might be expected if the solvents etched the surfaces by different amounts, but a break in the curvature is not apt to occur from this cause. Further evidence for the hypothesis of an ad-

sorbed layer lies in the fact that, without repolishing, a transition can be made from one curve to the other by cleaning with the different liquids.

Curve *C* is of particular interest. It was obtained by cleaning the crystals with either ethyl ether or petroleum ether then heating in air to a temperature just below red heat. The decrease in the loss of light at the surfaces is readily explained by the assumption that some volatile material was driven from the surfaces. It is hard to believe that heating to a temperature several hundred degrees below the melting point of the salt would alter the polish of the surface. Furthermore wetting the surfaces with the liquids used for cleaning brought the surface back to the state observed before heating. The discrepancy between the curve obtained for the heated specimens and the theoretical reflecting power may be partly due to an adsorbed layer which was not completely removed. In addition to this possible layer of material there must have been some diffuse scattering of light since a high powered microscope with dark field illumination showed that the surface was not completely free from scratches. There is also the possibility that the process of polishing disturbs the crystal structure near the surface sufficiently to alter the optical properties thereby increasing the loss of light at the surface. Since a small scratch on the surface may open a cleavage plane to a depth of several millimeters, it is conceivable that the cuts made by the polishing material might cause minute fractures near the surface. A broken layer of this type would scatter a considerable amount of light particularly at short wave-lengths, and, if the fissures were filled with air or some liquid, there might be an appreciable adsorption of light in the Schumann region.

Without direct measurements on the amount of light reflected by the lithium fluoride it is impossible to determine whether the change in the transmission of the surfaces with different methods of cleaning was due to an increase in the regular reflection or an increase in the absorption at the surface.

Glow discharge has been suggested as a method of cleaning surfaces. This method cannot be used on lithium fluoride because the discharge turns a surface layer of the crystal a deep red. Tests with a direct current discharge showed that both positive and negative ions were effective in producing this discoloration. The action of the ions is sufficiently weak that lithium fluoride may be used for windows on discharge tubes if there is sufficient separation between the crystal and the region of heavy ionization to prevent most of the ions from reaching the window.

Index of Refraction

The index of refraction was measured by crossing a 26° prism of lithium fluoride with a 25 cm concave grating in a vacuum spectrograph. It was found necessary to reduce the angular aperture of the grating to slightly less than two degrees to eliminate astigmatism and to increase the depth of focus. Furthermore this reduction in the aperture made the light passing through the prism sufficiently parallel to allow the grating to form the final image without the use of a second mirror. Thus the grating served the double purpose of monochromator and focusing device. Since the slit of the spectrograph was replaced by a 0.1-mm pinhole, the spectrum appeared as a row of dots on the photographic plate which was mounted perpendicular to the beam of light from the grating.

The prism placed between the grating and the photographic plate was arranged so that it could

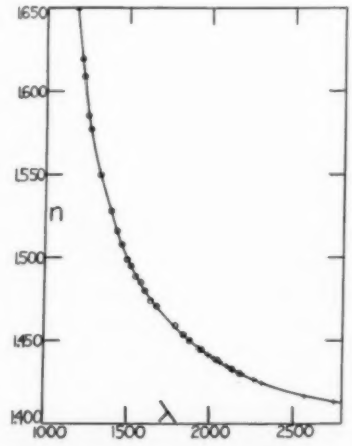

Fig. 3. Index of refraction.

be rotated about an axis passing through its center and parallel to the refracting edge. The prism could also be removed from the path of light without disturbing the vacuum. The introduction of the prism displaced the monochromatic images of the pinhole from their positions when formed by the grating alone. The tangent of the angle of deviation was then given by the ratio of this displacement to the perpendicular distance (20 cm) from the photographic plate to the prism.

When the prism was rotated about its center, each of the monochromatic images formed through the prism moved toward its corresponding undeviated image until the position of minimum deviation was reached. Rotating beyond this point caused the images to reverse their direction of motion. Thus rocking the prism through the positions of minimum deviation during the exposure gave a series of lines on the photographic plate. The ends nearest the spectrum formed by the grating alone represented the positions of the refracted monochromatic beams at minimum deviation. The index of refraction could then be expressed directly in terms of the angular displacement of the ends of these lines and the angle of the prism. Fig. 3 gives the index of refraction of lithium fluoride plotted against wave-length. The accuracy of the data is of the order of magnitude of three-tenths of one percent. As a check on the apparatus the values obtained for the index of refraction in the near ultraviolet were compared with those of Z. Gyulai.[5] His values are indicated by the crosses in Fig. 3.

Conclusions

Lithium fluoride in moderately thin pieces may be expected to transmit several percent of the light down to wave-lengths as short as 1100A, but this transmission cannot be assumed for a piece chosen at random. Impurities in the crystal, poor polish, and layers of foreign material on the surface may reduce the transmission in the Schumann region to a negligible quantity.

In conclusion the author wishes to express his gratitude to Professor T. Lyman for his continued interest in the progress of these experiments.

[5] Gyulai, Zeits. f. Physik **46**, 80 (1927).

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Intensity Distribution of the Continuous Spectrum of Hydrogen in Mixtures with Helium and with Neon*

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Shift of intensity in the continuous spectrum of hydrogen by admixture of rare gases. The continuous spectrum of hydrogen emitted in mixtures with helium and with neon was studied. The effect of these gases was to shift the maximum of intensity to longer wave-lengths. The ratio of intensity of the spectrum emitted in pure hydrogen to that emitted in the presence of helium as a function of the wave-length was determined by photographic photometry. In order to establish the reasons for this change in distribution of energy with wave-length in the continuous spectrum, a study of the intensity distribution of the many-line spectrum in the presence of helium was made. 20 cm of helium were sufficient to *reduce the vibration* in the excited states to practically zero. Hence the continuous spectrum emitted in the presence of helium must come from transitions from the *lowest vibrational level* of its upper state.

Measurement of intensity distribution of the continuous spectrum of hydrogen in the presence of helium. The wave-length distribution of energy in the continuous spectrum for 0.6 mm of hydrogen and 21 cm of helium was determined by comparison with the radiation emitted from the positive crater of the carbon arc, taken as the best available approximation of the ultraviolet black-body radiation. Under these conditions the maximum intensity of the continuous spectrum was in the neighborhood of 3200A in contrast with about 2500A for the spectrum under ordinary conditions. The position of this maximum is in agreement with the results of Finkelnburg and Weizel. The broadness of the maximum, however, does not agree with their results nor with the theoretical curves of James, Coolidge and Present. Another possible maximum noted at 4500A suggests that more than one electronic state contributes to the observed continuum.

* An extract from a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Harvard University.

I. Shift of Intensity in the Continuous Spectrum of Hydrogen by Admixture of Rare Gases

A. Problem and previous work

The present research was chiefly concerned with the molecular continuum of hydrogen in the near ultraviolet region. However, the intimate relation which exists between the continuum and the discrete molecular spectrum made it necessary to study the intensity relations in the discrete spectrum with a view to a clear understanding of the elementary processes involved in the radiation of the continuum.

Richardson,[1] using the experimental results of Merton and Barratt,[2] found that the addition of helium to a hydrogen discharge reduced the intensity of the high vibrational bands relative to the low vibrational bands. Roy[3] made further observations on the effect of helium which confirmed the findings of Richardson. Because of the low intensities involved and because increasing the pressure of helium did not produce an observable change in the effects, the highest pressure of helium employed by Roy was 20 mm. He also found that the intensity of the continuous spectrum was apparently increased with respect to the discrete spectrum.

In the theory of the hydrogen continuum proposed by Winans and Stückelberg[4] the spectrum originates in transitions from the $1s\sigma 2s\sigma\ {}^3\Sigma_g$ level as an upper state to the $1s\sigma 2p\sigma\ {}^3\Sigma_u$ level, discovered by Heitler and London,[5] as the lower state. The general mechanism of this theory has been established by the experiments of Finkelnburg and Weizel.[6] According to the theory we expect a change in the population of the vibrational levels of the upper state of the continuum to change the wave-length distribution of intensity. Thus the reduction of vibration by the addition of helium to a hydrogen discharge leads us to expect important changes in the continuous spectrum. The relative population of the vibrational levels of the upper state without helium present depends in a very complicated manner on the conditions in the discharge tube. The spectrum coming from each vibrational level can be calculated theoretically without too great difficulty. The relative magnitudes of these spectra, however, are unknown, for the population of the vibrational levels is unknown. Hence a quantitative check of the theory can be carried through only if the radiation originates from a definite vibrational level of the upper state. Then the excitation functions and other complicating relations need not be known.

Considerable progress has been made recently in the theory of the continuous spectrum. The *upper* level, its position and potential curve, is well known from the analysis of the discrete spectrum. For the *lower*, repulsive, level Winans and Stückelberg assumed a crude approximation. James and Coolidge[7] have developed new methods which enabled them to make great progress in the accurate computation of this level. If the vibration can be reduced to practically zero by the addition of high presssures of helium, the continuous spectrum under such conditions can be compared with the theoretical predictions. Consequently we studied the continuous spectrum emitted in the presence of helium and investigated the many-line spectrum in order to ascertain the effectiveness of helium in reducing vibration.

B. Experimental procedure

A discharge tube was constructed of Pyrex and arranged so that the capillary could be observed end-on from both ends. A thin window of Pyrex was blown on one end, and a thin quartz window was attached by means of a quartz-to-Pyrex graded seal to the other end so that simultaneous observations could be made with two spectrographs. A Hilger right-angle glass spectrograph was used to record the many-line spectrum situated largely in the visible, and a Hilger small quartz spectrograph was used to record the continuous spectrum in the ultraviolet. The electrodes were rolled from thin sheet

[1] O. W. Richardson, *Molecular Hydrogen and its Spectrum*, (Yale University Press, 1934).
[2] T. R. Merton and S. Barratt, Phil. Trans. Roy. Soc. **A222**, 369 (1922).
[3] A. S. Roy, Proc. Nat. Acad. Sci. **19**, 441 (1933).
[4] I. G. Winans and E. C. G. Stückelberg, Proc. Nat. Acad. Sci. **4**, 867 (1928).
[5] W. Heitler and F. London, Zeits. f. Physik **44**, 455 (1927).
[6] W. Finkelnburg and W. Weizel, Zeits. f. Physik **68**, 583 (1931).
[7] H. M. James and A. S. Coolidge, J. Chem. Phys. **1**, 825 (1933); H. M. James, A. S. Coolidge and R. D. Present, ibid. **4**, 187 (1936).

nickel and spot-welded so that they could be heated by an induction furnace to drive off occluded gases. The capillary, 3 mm inside diameter and 10 cm long, was platinized to enhance the molecular spectrum by aiding the recombination of hydrogen atoms. The tube was mounted in a water bath for cooling.

The tube was pumped out by means of a single-stage mercury diffusion pump backed by a Cenco Hyvac pump. The electrodes were raised to a bright red heat during pumping by means of an induction furnace, and the tube was baked out at 475°C.

Tank helium was purified by passing it slowly through activated charcoal at liquid-air temperature and then admitted to a misch metal arc. The helium was pure enough after passing through the charcoal trap to emit the helium bands quite strongly. However, the misch metal arc was run for ten or twelve hours before the helium was used.

Hydrogen, purified by diffusion through a hot palladium tube, was admitted to the discharge tube at a pressure of a few tenths of a millimeter. Exposures of varying times were then made with both the quartz and glass spectrographs. Pure helium was admitted at a few centimeters pressure, and another series of exposures on the same plates was made with the two instruments. The current was kept constant at 30 ma in both cases. Other plates were taken with the pressure of helium as high as 20 cm. Eastman ultraviolet sensitive plates were used with the quartz spectrograph while exposures made with the glass instrument were with Wratten type M panchromatic plates.

The same sort of experiment was then performed for mixtures of hydrogen and neon. The latter was purified by passing it through a charcoal trap in liquid air, but it was not so pure as the helium. Bands of OH and CO^+ were present. Also the line spectrum of neon was more strongly developed than that of helium in the previous case.

The ratio of the intensity of the continuous spectrum emitted by pure hydrogen to that emitted in the presence of helium was then determined. An enlarged image of the capillary of the discharge tube described above was focused by means of a quartz condensing lens on the slit of a quartz spectrograph. A series of exposures of constant time and current was made with the tube filled with hydrogen at 0.6 mm and 21 cm of helium. The intensity of each exposure was decreased a known amount from the preceding one by the interposition of wire gauze screens which had been calibrated with a photoelectric cell. Since the screens were not used in combinations, they were not shuffled as recommended by Harrison.[8] The tube was run from a 3000-volt rectifier with a large series resistance in the output circuit. Then, without disturbing the optical system, the tube was pumped out and filled with 0.6 mm of pure hydrogen. Several exposures were made with the current and time the same as for the first set of exposures and with the intensity reduced by very dense screens which had been calibrated by Dr. Rieke and kindly loaned for these experiments.

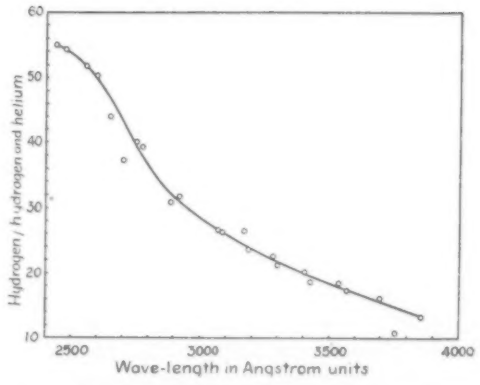

FIG. 1. Ratio of the intensity of the continuous spectrum in pure hydrogen at 0.6 mm to that in a mixture of 0.6 mm of hydrogen and 21 cm of helium, as a function of wave-length.

At each wave-length used, the relation between intensity and density of the photographic image was obtained from microphotometer readings taken from the first set of exposures. Then from these curves the ratio of intensity of the two continuous spectra was obtained from data taken from the second set of exposures. The ratio of the two intensities as a function of wave-length is shown in Fig. 1. With four exceptions the points fit a smooth curve nicely. An accuracy of better than 5 percent cannot be

[8] G. R. Harrison, J. O. S. A. & R. S. I. **18**, 492 (1929).

expected for a single measurement in photographic photometry. It should be pointed out, however, that in photometering a continuous spectrum we have the advantage of determining a smooth curve and not a definite number of points.

C. Results

These plates showed that the wave-length distribution of the continuous spectrum was changed to a marked degree by the addition of helium. The intensity in the longer wave-lengths was relatively enhanced (see Fig. 1). The discrete spectrum was greatly simplified—most of the energy being in the Fulcher bands in the yellow and red—but the dispersion was too small to allow a detailed study. Many lines of helium appeared on the plates, and at the highest pressures used a faint helium band appeared at 4648A. The results for neon were qualitatively the same as those found for helium.

Chalonge[9] has determined the relative changes in distribution produced by various pressures of hydrogen and various currents. Similar results have just been published by Gonsalves[10] in a report of an investigation of the conditions under which the hydrogen discharge may serve as a standard source for ultraviolet radiation. These changes are much less pronounced than the effects caused by the addition of helium.

D. Many-line spectrum in mixtures with helium

In order to interpret the shift of intensity reported, it was necessary to determine the extent of the modification of the vibration of the hydrogen molecule produced by the helium. For this purpose the discharge tube and auxiliary equipment were moved to a concave grating of ten-foot radius which gave a dispersion of about 5.2A/mm in the first order. Eastman type I–F plates were taken with the discharge in pure hydrogen and then with mixtures with various pressures of helium up to 40 cm.

It was found that the vibration was reduced to practically zero at pressures of helium of the order of 20 cm. Transitions ending on the $1s\sigma 2s\sigma\ {}^3\Sigma_g$ and the $1s\sigma 2p\pi\ {}^3\Pi_{ab}$ levels were studied in particular, for the $1s\sigma 2s\sigma\ {}^3\Sigma_g$ state is the upper level of the continuous spectrum. The ${}^3\Pi_{ab}$ state is of interest because it lies only 0.02 volt higher and combines with the ${}^3\Sigma_g$ state. Because of the small difference in energy between the states, transitions emitting radiation are extremely improbable. However, we expect radiationless transitions to occur—probably induced by collisions with other molecules or atoms.

These observations led to results which are of interest in connection with the many-line spectrum itself and which will be reported in another paper.

We are led to expect, in consequence of these results from the many-line spectrum, that the continuous spectrum emitted in the presence of helium arises largely from the *lowest vibrational level* of the $1s\sigma 2s\sigma\ {}^3\Sigma_g$ state.

II. Measurement of Intensity Distribution of the Continuous Spectrum of Hydrogen in the Presence of Helium

A. Experimental procedure

The result of the experiments described is of great interest for a quantitative comparison with the quantum theory. The ordinary electric discharge through hydrogen emits a continuous spectrum to which various vibrational levels contribute energy in an uncontrollable way. Only when the upper state is restricted to a definite vibrational level can a quantitative test of the theory be carried through.

The principal difficulty is due to the lack of a standard source for comparison of intensities in the ultraviolet. At the temperature easily available in the laboratory, black-body radiation is vanishingly small. No calibrated tungsten lamp was available. However, the positive crater of a carbon arc emits radiation that corresponds over a range of at least 1000A to a definite black-body temperature. Henning and Heuse[11] measured the temperature at two wave-lengths, 6560A and 5450A, and found it to be constant at 3703°K. Recently Chaney, Hamister, and Glass[12] of the National Carbon Co. determined the arc tem-

[9] Daniel Chalonge, Ann. de physique **1**, 123 (1934).
[10] V. E. Gonsalves, Physica **2**, 1003 (1935).
[11] F. Henning and W. Heuse, Zeits. f. Physik **32**, 799 (1925).
[12] N. K. Chaney, V. C. Hamister and S. W. Glass, Trans. Electrochem. Soc. **67**, 201 (1935).

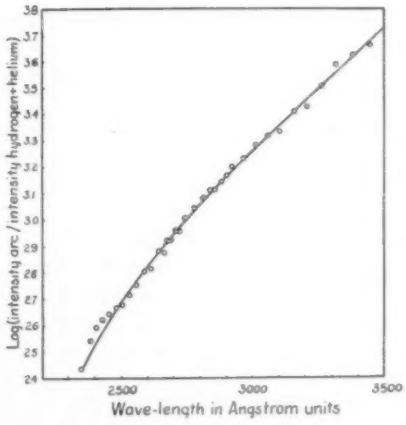

FIG. 2. Logarithm of the ratio of intensity of the carbon arc to the continuous spectrum with 0.6 mm of hydrogen and 21 cm of helium, as a function of the wave-length.

perature to be 3810°K. For the present experiment an arc with the same type of carbon was used as a standard.

In order to prevent fogging of the plate by scattered visible light two filters were used, a Corning red-purple Corex glass filter for the range from 4000A down to 2800A and a filter of bromine vapor down to 2400A.

An aluminum mirror on a turntable reflected the light of either the arc or the capillary into a quartz lens which formed an image of the source on the slit of a quartz spectrograph. The calibrated screens and filter were placed immediately in front of the lens.

The discharge tube, filled with 0.6 mm of hydrogen and 21 cm of helium, was run at 80 ma on a 3000-volt rectifier until quite steady. Only exposures of constant time were compared. For this purpose intensities were reduced by calibrated screens and a rapidly rotating sector.

From a set of exposures with the discharge tube and calibrated screens, the curves showing the relation of intensity and density at each wave-length were determined with a microphotometer. From these and the exposures of the arc, the relative intensity of the two sources was then obtained as a function of the wave-length.

B. Results

The logarithm of the ratio of intensity is plotted as a function of the wave-length in Fig. 2.

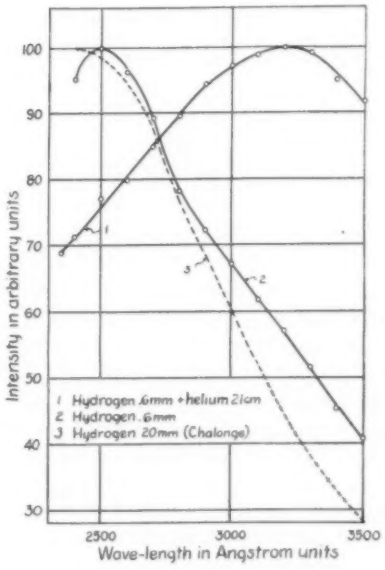

FIG. 3. Curves showing intensity of the continuous spectrum as function of wave-length.

The values were obtained from two plates well overlapped as to the wave-length ranges. One plate covered the range between 3500A and 2650A, while the other extended from 2900A to 2350A. The points fit the smooth curve shown except in the region below 2500A. In this region there is a strong carbon line along with some faint lines of silicon. Consequently we may consider the curve below 2480A as an extrapolation.

By comparison with the spectrum from the arc, the intensity distribution with wave-length of the helium-hydrogen mixture was calculated. These values are plotted in Fig. 3 with the maximum intensity arbitrarily set at 100. By using the values for the ratio of intensities of the spectrum of pure hydrogen and of hydrogen and helium, the intensity distribution for pure hydrogen was obtained. This relation is also shown in Fig. 3, again with the maximum intensity arbitrarily set at 100. The dotted curve is the one found by Chalonge[9] for a discharge in pure hydrogen at 20 mm. The smoothness of these curves arises from the fact that they were determined from other smooth curves, and hence

they are not indicative of the experimental error. These curves rest on the data shown in Fig. 1 and Fig. 2.

C. Discussion of results

The random error exhibited in the curves is satisfactorily small. We must look carefully, however, for possible sources of error which are functions of the wave-length.

It should be noted that the slit of the spectrograph was not illuminated in the same way by the images of the capillary and of the crater of the arc, for the image of the arc is sensibly a disk and that of the capillary is a truncated cone. This difference of image shape might well cause an incorrect determination of the ratio of intensities, but it is not likely to cause an error which varies appreciably with the wave-length.

The sector used to reduce the intensity was rotated at 1800 revolutions per minute. From the work of Webb[13] and a discussion by Harrison[14] we expect the error in the reduction of intensity at this speed to be small. Moreover it is the way in which this error varies with wave-length that concerns us. Hence if a small error does occur, it is a second-order effect and will not disturb our measurements.

An obvious source of uncertainty is the use of the arc as a black-body, for we are extrapolating the properties of the arc from the visible into the near ultraviolet. An increase in the temperature of the arc would shift the position of the maximum for the hydrogen continuum toward the shorter wave-lengths. A small error of this sort would not, however, affect the essential character of the results. The deviation of the carbon spectrum from the black-body spectrum seems the main source of error which cannot well be estimated as long as no reliable standard source is available.

We believe that the observed continuum cannot be appreciably due to the helium molecule because of the low excitation potential of the hydrogen continuum—11.8 volts compared with about 20 volts for helium.

The intensity curve for pure hydrogen at 0.6 mm is in fair agreement with Chalonge's curve for 20 mm and is deformed in the way he predicts for the difference in pressure.

The position of the maximum at 3200A for the hydrogen-helium mixture is in agreement with the work of Finkelnburg and Weizel, but the broadness is not. Also this broadness does not agree with the theoretical predictions of James, Coolidge and Present.[15] If electronic states other than $1s\sigma 2s\sigma\ ^3\Sigma_g$ give rise to continua in the observed range, the curve shown in Fig. 3 should not be compared with the theoretical predictions. On the plates taken with mixtures of helium and of neon what appeared to be another maximum of intensity in the continuous spectrum was observed at about 4500A. However, the appearance of a maximum might be caused by the varying sensitivity of the plate. This maximum is in the region where Brasefield[16] found a new continuum in an electrodeless discharge at low pressures of hydrogen and is possibly related to it. The high excitation potential of helium would allow a large population in the high electronic states of the hydrogen molecule.

The principal result of the experiments reported here is that helium and neon by reducing vibration produce a major change in the continuous spectrum of hydrogen. Under these conditions a large portion at least of the continuum arises from the lowest vibrational level of the $1s\sigma 2s\sigma\ ^3\Sigma_g$ state.

The author wishes to express his sincere appreciation to Professor O. Oldenberg for his interest in the work and for his numerous helpful discussions and suggestions.

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A Note on the Intersystem Combination Lines in A III

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Intersystem combination lines of the third spectrum of argon have been discovered in the extreme ultraviolet. The lines are listed and the singlet term values previously published are revised to conform to the same basis as that of the triplet terms. Wave-lengths of the "forbidden" lines of this stage of ionization of argon have been calculated. One of them is in close agreement with a nebular line at λ7135.6.

IN a term table for A III already published[1] the singlet terms have been listed on the assumption, following Bowen,[2] that the $s^2p^4\ {}^1D_2$ term lies at 14,000 cm^{-1} above the lowest $(s^2p^4\ {}^3P_2)$ term of the atom. In a note appended at proof mention was made of additional triplet terms discovered by de Bruin[3] and the term table was amended to include them. Of these the $(^2D)3d\ {}^3F^0$ term gives no combinations with the ground triplet but has since been discovered to give intersystem combinations with the low $s^2p^4\ {}^1D_2$ term. Following this clue other intersystem lines have been found so that now the values of the singlet terms can be given on the same basis as that of the triplet terms. Tables I and II list these intersystem lines and the revised values which they give for the singlet terms previously published. These tables should be taken as supplementing and correcting Tables IV and V of the earlier article.[1] While the values of the suggested $(^2D)3d\ {}^1F_3^0$ and $(^2D)4s\ {}^1D_2^0$ terms have been shifted to conform to the new scale, the reality of these terms remains open to considerable question.

TABLE I. *Intersystem combination lines in A III.*

λ	INT.	ν	CLASSIFICATION
1205.95	1	82,922.5	$s^2p^4\ {}^1S_0 - sp^5\ {}^3P_1^0$
1002.095	3−	99,790.9	$s^2p^4\ {}^1D_2 - sp^5\ {}^3P_2^0$
699.72	1−	142,907	$s^2p^4\ {}^1D_2 - (^4S)3d\ {}^3D_2^0$
623.767	5	160,316	$s^2p^4\ {}^3P_0 - sp^5\ {}^1P_1^0$
579.212	3	172,648	$s^2p^4\ {}^1D_2 - (^3D)3d\ {}^3F_2^0$
578.386	4	172,895	$s^2p^4\ {}^1D_2 - (^3D)3d\ {}^3F_2^0$

TABLE II. *A III singlet term values.*

$(3s)^2$	$(3p)^4$	$^3P_2$	0
$(3s)^2$	$(3p)^4$	$^1D_2$	14,010
$(3s)^2$	$(3p)^4$	$^1S_0$	31,873
$3s$	$(3p)^5$	$^1P_1^0$	161,886
(^2D)	$3d$	$^1F_3^0$	179,531 ?
(^2D)	$4s$	$^1D_2^0$	200,318 ?

These term values permit the more exact calculation of the wave-lengths of the "forbidden" lines due to A III and locate them as follows:

$$^3P_2 - {}^1D_2\ \lambda 7135.7, \quad {}^3P_1 - {}^1D_2\ \lambda 7751.0,$$
$$^1D_2 - {}^1S_0\ \lambda 5596.6, \quad {}^3P_1 - {}^1S_0\ \lambda 3250.$$

The first of these predictions agrees very closely with the line observed by Merrill at λ7135.6[4] in NGC 6572 and NGC 7027. This line has been so identified by Bowen[2, 5] upon the basis of interpolation along an isoelectronic sequence. Bowen's suggestion for λ7135.6 is considerably strengthened by the present results, but a final decision in the interpretation of the nebular line must await observations to detect its companion line at λ7751. This line lies further into the infrared than nebular observations have yet been extended. λ5597 and λ3250 would be expected to be observed only in such celestial objects as favor the excitation of the "auroral" type of "forbidden" line,[6] namely, certain stages of novae and a few nebulae such as NGC II 4997.[7] Observations in the proper spectral ranges are unfortunately scarce.

Opportunity is taken to correct a misprint in Table V of the earlier article.[1] The value of the $(^2P)3d\ {}^3P_2^0$ term should be 213,945.

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Effects of Exchange Forces on the Excitation Function of Li^7 Under Proton Bombardment

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In a recent paper Van Vleck gave an approximate method of estimating the effective potential due to Majorana exchange forces, which acts on a proton or neutron when they collide with a nucleus. The effective potential obtained in this manner varies with velocity. Calculations reported on here show that the effect of this variation with velocity on the shape of the excitation curve of lithium under proton bombardment is small and is of no practical importance. The position of stationary proton levels which are consistent with the excitation curve is affected seriously. Agreement of calculations with the mass defect of Be^8 is materially improved through the use of the exchange potential. With a radius 0.35×10^{-12} cm the empirical mass defect of Be^8 is accounted for perfectly by the calculation which simultaneously fits the excitation curve for the 8-cm particles.

IN a recent paper Van Vleck[1] using the Majorana exchange operator and assuming together with Feenberg[2] a neutron proton interaction of the form Ae^{-ar^2} made calculations of the cross section of heavy nuclei for collisions with slow neutrons. In this work he developed a simple approximate method capable of representing the exchange forces by an equivalent central field which varies with the velocity of the neutrons. Van Vleck's considerations can be applied to protons incident on nuclei simply by interchanging protons and neutrons in his formulas. Calculation by means of them shows that the mean attraction exerted by a nucleus on an incident proton or neutron increases as the velocity of these particles decreases. Similarly a particle in a bound state can be expected to be subjected to a stronger attractive potential than an incident particle. Van Vleck's considerations are only approximate inasmuch as they involve a statistical treatment of particles constituting the nucleus as well as the usual approximations necessary for the reduction of a many body problem to Hartree-Fock fields. They are besides supposed to apply only to large nuclei inasmuch as the surface effects of v. Weizsäcker[3] are neglected. Nevertheless one may suppose that in order of magnitude the difference between the effects of using Wigner and Majorana forces can be estimated also for light nuclei by means of Van Vleck's formulas.

Calculations on the excitation curve of $Li^7+H^1\rightarrow He^4+He^4$ were published recently.[4] In these the field acting on the proton was supposed to be independent of velocity. It was thought of interest to estimate the effect of using Van Vleck's potential on the same problem. One may expect two kinds of effects to arise from the velocity dependence of Van Vleck's potential. Firstly, the excitation functions change and secondly the stationary levels of the protons in the nucleus are affected.

The effect on the excitation curves is usually small and may be expected to be of importance only in those exceptional cases in which the excitation curve depends very critically on the position of a stationary or virtual level. The primary reason for this is the fact that the changes in kinetic energy of incident protons with which one is concerned (1 MEV) are small in comparison with the depth of the stationary levels which are of interest ($\sim$20 MEV). Also the change in the attractive potential is itself smaller than the change in kinetic energy with which it is associated. Thus the main modification due to using the exchange potentials consists in depressing the stationary proton levels which one expects for a given shape of the excitation curve.

As in the previous calculation[4] it is supposed that the chance of disintegration is determined by the probability of finding the incident proton in the nucleus so that the number of disintegrations per second is given by P times the average

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[2] E. Feenberg, Phys. Rev. **47**, 850 (1935).
[3] C. F. v. Weizsäcker Zeits. f. Physik **96**, 431 (1935).
[4] M. Ostrofsky, G. Breit and D. P. Johnson, Phys. Rev. **49**, 22 (1936). This paper will be referred to as (I).

number of incident protons located in the nucleus. The quantity P is taken to be independent of the velocity of the protons.

In agreement with the notation used previously[4] we let U stand for the negative of the potential inside the nucleus so as to have $U>0$. According to Van Vleck

$$U=-A[E(w_+)\mp E(w_-) +2(\alpha/\pi K_p^2)^{\frac{1}{2}}(e^{-w_+^2}-e^{-w_-^2})], \quad K_p\gtrless K \tag{1}$$

$$w_\pm=\frac{|K_p\pm K|}{2\alpha^{\frac{1}{2}}}, \quad E(w)=2\pi^{-\frac{1}{2}}\int_0^w e^{-t^2}dt,$$

$$K=\left[\frac{9\pi N}{4r_0^2}\right]^{\frac{1}{2}}.$$

Here N is the number of neutrons in the nucleus, K_p is 2π times the wave number of the proton when it is in the nucleus and the remaining symbols are defined by the equation itself. The sign before $E(w_-)$ should be taken as $-$ or $+$ depending on whether $K_p>K$ or whether $K_p<K$. From the conservation of energy it follows that

$$\mu T/m=0.0\ 2\ 4\ 8\ 3\ mZ^2Z'^2(K_pa)^2-U, \tag{2}$$

where m is the mass of the proton, μ is the reduced mass, T is the kinetic energy of the proton in the coordinate system in which the nucleus is at rest, while Z and Z' are, respectively, the atomic numbers of the proton and the nucleus. The unit of energy in this equation is the MEV. The quantity a stands for $\hbar^2/\mu ZZ'e^2$. This equation is convenient because K_pa is analogous to the parameter $1/\eta=$"1 3 7" $(v/c)/ZZ'$ which enters into the computation of wave functions in the Coulombian region outside the nucleus. Eqs. (1) and (2) determine U as a function T. Substitution of numbers shows that this relation is nearly linear and that U decreases as T increases. Thus the slower the proton the more effectively it is attracted by the nucleus. With U known as a function of T the collision cross section σ can be calculated in the same way as was done in I. The calculations are summarized in Table I. The values of σ and U obtained by using Eq. (1) are called σ_{ex}, U_{ex} in Table I so as to indicate that they correspond to the use of exchange forces. The second and third columns in Table I correspond to incident protons of zero angular momentum ($L=0$) while the fourth and fifth columns correspond to protons with angular momentum $\hbar$ ($L=1$). The values of A and α were so chosen as to give a good agreement between the theoretical and the experimental yield curves. It will be noted that the values of U_{ex} which correspond to this condition are practically the same as those found previously. All of the calculations were made for a nuclear radius of 0.347×10^{-12} cm. The values of U in Table I correspond to $A=-194.87$ MEV, $\alpha^{-\frac{1}{2}}=1.6\times10^{-13}$ cm for $L=0$ and to $A=-102.43$ MEV, $\alpha^{-\frac{1}{2}}=1.5\times10^{-13}$ cm for $L=1$. This choice of A and α was made rather arbitrarily by assigning a reasonable α and adjusting A to give the right U for $T=0$.

TABLE I. *Calculations of collision cross sections.*

	$L=0$		$L=1$	
T in MEV	U_{ex} in MEV	σ_{ex}/σ	U_{ex} in MEV	σ_{ex}/σ
0	35.00		22.00	
35.4	34.990	1.023	21.990	0.931
56.1	34.980	1.022		
89.0	34.960	1.023	21.975	0.949
141.0	34.940	1.015		
223.5	34.910	1.010	21.940	0.978
354.2	34.865	1.000	21.905	1.000
561.3	34.785	0.984	21.850	1.027
890.0	34.665	0.958	21.764	1.057
1410	34.465	0.916	21.625	1.090
2235			21.406	1.122

The way in which σ_{ex}/σ varies with T in Table I can be understood as the result of progressive shifts in the position of resonance and stationary levels. The importance of such levels for the excitation function was already discussed in I. By a resonance or stationary level in the explanation given below we mean for any T the level which we would have if the value of U_{ex} corresponding to that T were kept fixed for all T. This way of talking about the resonance levels is rational in this connection because the position of the level matters through its effect on the resonance denominator in the formula for σ; the value of the resonance denominator, however, depends only on properties of the wave function at a given energy, i.e., only on U_{ex} at that energy. Proximity to a resonance or stationary level increases the yield. For protons with $L=0$ and $U\sim35$ MEV the closest level is a resonance level located roughly at $T=+10$ MEV. As T increases, the value of U_{ex} decreases, the reson-

TABLE II. *Calculations on position of stationary levels.*

	$L=0$				$L=1$			
	$l=0$		$l=1$		$l=0$		$l=1$	
	U	W	U	W	U	W	U	W
VAN VLECK POTENTIAL	56.42	−42.85	42.98	−17.13	27.09	−15.45	22.38	−1.25
CONSTANT POTENTIAL	35.00	−22.55	35.00	−10.51	22.00	−10.90	22.00	−1.00
	$\Delta W=-20.30$		$\Delta W=-6.62$		$\Delta W=-4.55$		$\Delta W=0.25$	
	$\Delta W'=-19.84$		$\Delta W'=-6.50$		$\Delta W'=-4.40$		$\Delta W'=0.24$	

ance level is moved up and therefore away from the experimental region and consequently σ_{ex}/σ decreases. For protons with $L=1$ the closest level is a stationary level located approximately at -1.5 MEV. As T increases, this level also moves up and therefore this time towards the experimental region. As a result σ_{ex}/σ increases with T.

As has been discussed in (I) the approximate mass of Be^8 can be used as an argument in favor of fitting the excitation function of $L=0$ of the incident protons and using the corresponding value of U. This argument still holds as will be seen from calculations on the positions of stationary levels using Van Vleck's potential. While in (I) the depth of the potential well U was kept constant, we now keep A and α constant and equal to the values which give good fits to the experimental excitation function.

In Table II are given results of calculations on the position of stationary levels using the exchange as well as the constant potentials. Exact radial functions were used inside the "well" and WKB approximations outside. The calculations were checked by using first-order perturbation formulas, and computing the energy difference between corresponding levels with the same L for the exchange potential and the constant potential. The difference between U_{ex} and U was used as the perturbation potential and the wave functions outside the "well" were approximated by ae^{-br}.

In Table II L represents the angular momentum of the incident proton, l the angular momentum associated with the stationary level, while W stands for the energy of that level. The difference in depths of corresponding stationary levels between the Van Vleck and constant "wells" is denoted by ΔW while $\Delta W'$ is the same obtained from the perturbation calculation. All the numerical values are in MEV.

It will be noted that for $L=1, l=1$, the change due to using exchange forces rather than a constant potential is insignificant. It is thus still impossible to use this well for obtaining the value $W=-17$ MEV required by the mass defect of Be^8. On the other hand, the value -17.13 for $L=0$, $l=1$, using Van Vleck's potential, agrees very well with what is needed and is seen to be in better agreement with experiment than -10.5 obtained by using the constant potential. The apparently excellent agreement thus obtained is probably somewhat fortuitous. It is nevertheless encouraging that the agreement is made better rather than worse through the use of exchange forces. The value of A which is needed to account for the large U required for $L=0$ is larger than the values needed to account for the mass defects of light nuclei. It agrees better with the values which follow from mass defects of heavy nuclei using statistical considerations as may be expected from the fact that Eq. (1) was also obtained by a statistical method. Thus presumably the value of A used above is too high and a more accurate calculation will give the same potential for a smaller A. This may also be expected from the fact that from the mass defects of light nuclei the statistical method gives a too large estimate of A as compared with more exact methods of calculation.[5]

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The General Theory of Fluctuations in Radioactive Disintegration*

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The chance, $W_n(0, t)$, that n atoms of a radioactive source will disintegrate in the interval $(0, t)$ may be found if we know $f_r(t)dt$, the chance that a disintegration will occur in dt at t after r have occurred in $(0, t)$. Bateman's differential equation is generalized to cover the case in which f_r depends on t and r. The solution is given for the case in which it depends only on t. A detector of efficiency g subtends a solid angle $4\pi A$ at a source (decay constant λ), containing N atoms at time zero. The probability of n *counts* in the interval (T_1, T_1+T_2) is

$$P_n(T_1, T_1+T_2)=C_n^N(e^{\lambda T_2}-1)^n e^{-N\lambda(T_1+T_2)} \times(gA)^n[1+e^{\lambda T_2}(e^{\lambda T_1}-1)+(1-gA)(e^{\lambda T_2}-1)]^{N-n}.$$

Putting g and A equal to 1 we get the probability $W_n(T_1, T_1+T_2)$ of n disintegrations in this interval. Bortkiewicz's formula $W_n(0, t)=C_n^N(e^{\lambda t}-1)^n e^{-N\lambda t}$, is a special case. If we measure a great number, r, of intervals between the disintegrations of atoms in a single source, *the fraction of the intervals that exceed* t has the "expected" value

$$(e^{-N\lambda t}-e^{-(N-r)\lambda t})/r(1-e^{\lambda t}).$$

The problem of determining fluctuations in the disintegration of a single substance from fluctuations in counting is solved for the case in which each disintegration gives a single ray capable of actuating the counter. Fluctuations in counts produced by two or more independent sources are considered. Since gamma-rays and secondary beta-rays are not emitted by every disintegrating atom, the distribution of counts due to such rays is discussed. The effect of the recovery time, τ, of the counter is discussed, using at first the assumption that τ is the same for all counts, an approximation useful at low counting rates. With a source that would produce f counts per sec. if τ were zero, the probability of n counts in the interval $(0, t)$ is obtained for two cases: (1) The counter is not clogged at time zero; (2) it is clogged. The second case has practical interest; the probability of an interval greater than t between counts is 1 if $t\leqq\tau$ and $e^{-f(t-\tau)}$ if $t\geqq\tau$. Differential-difference equations for the fluctuation-functions are derived. The fluctuations of counts due to a constant source, in a counter with *variable recovery time*, are obtained, using Skinner's formula for the frequency distribution of recovery times. For t values greater than the maximum recovery time, the probability of an interval greater than t is $e^{-f(t-\tau')}$; τ' is a constant. Formulas are derived for the *stock fluctuations* of each substance in a source containing several members of a radioactive series, subject to any desired initial conditions; recurrence equations governing the stock probabilities are given. The probability of a stock n of the daughter of a constant parent which yields f disintegrations per sec. is $S_n=(f/\lambda)^n e^{-(f/\lambda)}/n!$; here λ is the decay constant of the daughter. General methods for finding the fluctuations in the emission of an entire radioactive series, or any part of a series, are given. *The disintegration-fluctuations of the daughter of a constant parent obey the formula* $W_n=(ft)^n e^{-ft}/n!$ which applies also to the parent. However, fluctuations of parent and daughter are coupled, so that the Bateman type of formula does not apply to their *combined* emission. Instead, the probability of an interval greater than t is $\exp[-ft-(f/\lambda)(1-e^{-\lambda t})]$.

A. FLUCTUATIONS IN THE DISINTEGRATION OF A SINGLE SUBSTANCE

1. A generalization of Bateman's differential equation

IN 1910 Bateman[1] derived his famous formula for the probability that n atoms will disintegrate in time t, in a source whose diminution during the experiment can be neglected. It is

$$W_n=(ft)^n e^{-ft}/n!, \qquad (1)$$

where f is the average number of disintegrations per unit time. Bortkiewicz[2] gave the formula which applies to a decaying source. We shall present a differential equation which covers a much broader range of possibilities, yielding the formulas of Bateman and Bortkiewicz as special cases. Suppose we know $f_r(t)dt$, the probability that one disintegration will occur in dt at t when r have occurred in the interval $(0, t)$. Then there are two cases to be considered:

A. f_r is a function of t or a constant, but does not depend on r, so that we may omit the subscript.

B. f_r depends on both r and t.

* Preliminary communication, see Phys. Rev. **48**, 772 (1935).

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[2] Bortkiewicz, *Die Radioaktive Strahlung als Gegenstand wahrscheinlichkeitstheoretischer Untersuchungen* (Springer, 1913), p. 75.

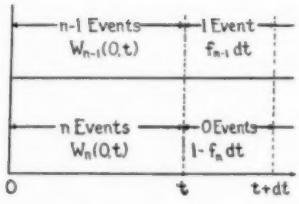

FIG. 1. Two independent ways in which n events may occur in time $t+dt$.

In each case the primary problem is to find $W_n(0, t)$, the probability that n disintegrations occur in the interval $(0, t)$. When this is known we can obtain other useful functions, such as $W_{s,n}(T_1, T_1+T_2)$, the probability that n events occur in the interval from T_1 to T_1+T_2 when s events are known to have occurred in the interval $(0, T_1)$. We shall drop the time-arguments when they are not needed for clarity.

To obtain a differential equation obeyed by $W_n(0, t)$, we note that the probability of n events between 0 and $t+dt$ is the sum of two terms (Fig. 1): the probability of $n-1$ events in the interval $(0, t)$, times the probability of one event in dt at t; (2) the probability of n events in the interval $(0, t)$, times the probability of no events in dt at t. Thus,

$$W_n(0, t+dt) = W_{n-1}(0, t)f_{n-1}dt + W_n(0, t)\cdot(1-f_n dt);$$

or

$$dW_n/dt = f_{n-1}W_{n-1} - f_nW_n; \quad n=1, 2, \cdots, \tag{2}$$

When n is zero, f_{n-1} is zero, so that

$$dW_0/dt = -f_0W_0. \tag{2a}$$

These equations are a generalization of the Bateman equations, reducing to them when f is constant. The initial conditions are $W_0(0, 0)=1$; $W_n(0, 0)=0$ when n is not zero. Therefore

$$W_0 = e^{-\int_0^t f_0 dt}, \tag{3}$$

and we can obtain the other W's in succession by using the general solution of (2), namely,

$$W_n = e^{-\int f_n dt}\cdot\int_0^t f_{n-1}W_{n-1}e^{\int f_n dt}dt; \quad n=1, 2, \cdots. \tag{4}$$

2. General solution when f is a known function of t

If the chance $f(t)dt$ of an event in time dt at t depends only on t, Eq. (2) becomes

$$dW_n/dx = W_{n-1} - W_n, \tag{5}$$

wherein

$$x = \int_0^t f(t)dt. \tag{6}$$

The solution of (5) is

$$W_n(0, t) = x^n e^{-x}/n! \tag{7}$$

Also, we have

$$W_n(t_1, t_1+t) = x^n e^{-x}/n!, \tag{8}$$

where now

$$x = \int_{t_1}^{t_1+t} f(t)dt.$$

The mean number of events in the interval t_1, t_1+t is found to be x, and the standard deviation (r.m.s. fluctuation about the mean) is $x^{\frac{1}{2}}$. The Lexian ratio,—that is (standard deviation)^2/mean,—is therefore unity, as it also is for a source of constant strength. Eqs. (7) and (8) give approximate information about the fluctuations of a preparation containing a large number of atoms, where variations of the stock are negligible. For a preparation whose decay constant is λ, containing N atoms at time zero, f approaches $Ne^{-\lambda t}\lambda$ as N increases. Putting $x=N(1-e^{-\lambda t})$ in Eq. (7), we may compute approximate values of the fluctuation of ionization current, and other useful quantities. Eqs. (7) and (8) cannot be applied to a source that contains only a small number of atoms and has a decay constant so high that the strength of the source falls off considerably during the experiment. For such a source, the more general Eq. (2) must be used.

3. The case of a decaying source

Probability of n counts in a definite interval beginning at a definite time. If a source contains N atoms at time 0, and n disintegrate in time t, the probability that one of the remaining atoms will disintegrate in t, $t+dt$ is

$$f_n dt = (N-n)\lambda dt. \tag{9}$$

Then the solution of (2) is

$$W_n(0, t) = C_n{}^N(e^{\lambda t}-1)^n e^{-N\lambda t}, \tag{10}$$

where $C_n{}^N = N!/(N-n)!n!$. A simple way to

show this is the following. If we observe the source continuously during the interval $(0, t)$ we are really making N experiments, each one of which consists in seeing whether a particular atom disintegrates. The chance that a particular atom will disintegrate is $p \equiv 1-q = 1-e^{-\lambda t}$, and the probability of n disintegrations in N "trials" is $C_n{}^N p^n q^{N-n}$ which may be converted into (10). If we study a great number of identical sources, the "expected" average of n is pN, or $N(1-e^{-\lambda t})$, and the standard deviation is

$$(Npq)^{\frac{1}{2}} = [N(1-e^{-\lambda t})e^{-\lambda t}]^{\frac{1}{2}}.$$

Let us now obtain $W_n(T_1, T_1+T_2)$, the probability that n events will occur in the interval (T_1, T_1+T_2) when we do not know the number that occurred in the previous interval $(0, T_1)$. By Eq. (10), the probability that n events occur in (T_1, T_1+T_2) when s have occurred in $(0, T_1)$ is

$$W_{s,\,n}(T_1, T_1+T_2) = C_n{}^{N-s}(e^{\lambda T_2}-1)^n e^{-(N-s)\lambda T_2}. \quad (11)$$

Therefore,

$$W_n(T_1, T_1+T_2) = \sum_{s=0}^{N-n} W_s(0, T_1) W_{s,\,n}(T_1, T_1+T_2)$$

$$= C_n{}^N (e^{\lambda T_2}-1)^n e^{-N\lambda(T_1+T_2)} \sum_{s=0}^{N-n} C_s{}^{N-n} [e^{\lambda T_2}(e^{\lambda T_1}-1)]^s$$

$$= C_n{}^N (e^{\lambda T_2}-1)^n e^{-N\lambda(T_1+T_2)} [1+e^{\lambda T_2}(e^{\lambda T_1}-1)]^{N-n}. \quad (12)$$

The average number of disintegrations and the standard deviation may be obtained from Eqs. (18a) and (18b), as special cases.

Methods for testing fluctuation theory. Eq. (12) can be used to test the fluctuation theory for substances of any desired half-life, whereas previous work has dealt with approximately constant sources. The appropriate procedures for experimentation and for summarizing the data are somewhat different from those employed in the past. When dealing with a constant source, one obtains chronograph records showing the times of occurrence of a great number of disintegrations, and then two convenient methods of preparing the data for statistical analysis are available, associated with the names of Marsden and Barratt, and of Rutherford and Geiger, respectively. They are as follows:

(1) The fraction $F_0(t)$ of the intervals between disintegrations that are longer than t is determined for various values of t. These fractions should obey the law $F_0(t) = e^{-\lambda t}$, according to Eq. (1).[3]

(2) The record is divided into equal intervals of length t, and the fraction of the intervals containing n events is determined, for different values of n.

In dealing with decaying substances, several plans are available. One may obtain chronograph records from a number of sources, all of the same initial strength as nearly as possible. Then, referring to Eq. (12), one may fix the value of T_2 and find the *fraction of the sources* that give n particles in the interval $(0, T_2)$; n particles in the interval $(T_2, 2T_2)$; and so on, taking $T_1 = 2T_2$, $3T_2$, etc., in succession. The process can be repeated for other values of T_2, and the dependence of the results on n, T_1, and T_2 can then be compared with Eq. (12). This method is obviously desirable when one must deal with weak sources that decay rapidly, so that only a limited number of counts can be obtained from each source. On the other hand, when sufficiently strong preparations are available, one will desire to analyze records obtained from a single source.

[3] This statement is often questioned by those who meet it for the first time. Eq. (1) with $n=0$, really gives the probability that zero events will occur in a time t after an *arbitrary* initial instant. Therefore it seems unfair in analyzing data, to make every interval begin at the instant when a disintegration occurs. This mental difficulty may be dispelled by noting that the probability λdt for an event to occur in dt is not affected by the occurrence or nonoccurrence of an event at the beginning of dt. The constant source may be considered as one having an "infinite" stock of atoms and an "infinitesimal" decay constant.

Assigning to T_2 a fixed value t, one counts the number of particles in the intervals 0, t; t, $2t$; etc., as before and determines the *fraction of the intervals* in which n particles occur. Let there be r intervals altogether. In the jth interval, the probability of n particles is $W_n([j-1]t, jt)$. For fixed values of n and t, these probabilities depend only on j, and by a known theorem,[4] this means that *out of r intervals the fraction containing n particles* has the expected value

$$r^{-1}\sum_{j=1}^{r} W_n([j-1]t, jt)$$

$$=r^{-1}C_n{}^N(e^{\lambda T_2}-1)^n\sum_{j=1}^{r}e^{-jN\lambda T_2}(1-e^{\lambda T_2}+e^{j\lambda T_2})^{N-n}.$$

Unfortunately this sum cannot be simplified, as one may see by examining the trivial case $r=2$. For practical purposes, it is necessary to use another method of analysis, now to be explained.

Size-distribution of intervals between emissions from a single source. It is possible to find a formula for the fraction $F_0(t)$ of the intervals between emissions that exceed t. Let the source contain N atoms initially. By Eq. (10), the probability that the first interval will exceed t is $e^{-N\lambda t}$. After the first disintegration has occurred, the probability that the second will exceed t is $e^{-(N-1)\lambda t}$, and so on. For a given value of t, each of these probabilities is fixed, so that out of r intervals the expected number exceeding t is $\sum_{j=1}^{r} e^{-(N-j+1)\lambda t}$; that is, *the fraction of the r intervals that exceed t has the expected value*

$$F_0=(e^{-N\lambda t}-e^{-(N-r)\lambda t})/r(1-e^{\lambda t}). \qquad (13)$$

This generalization of Bateman's formula makes it possible to carry out a Marsden-Barratt analysis of the fluctuations of a single decaying source.

B. Fluctuations in Counting with a Detector of Negligible Recovery Time

4. Effect of solid angle subtended at source by the detector, and of detector efficiency

So far we have dealt with the time-distribution of all the disintegrations occurring in a source. A direct test of the formulas derived would require a detector of perfect efficiency, which catches all the emitted particles. Therefore we now discuss the influence of the solid angle subtended by the detecting device, and of its failure to record all particles that reach it. To bring out clearly the effects of these factors, we assume in Sections 4 to 6 that the finite time-resolving power of the instrument can be neglected, which means that the results in these sections are valid only for low counting rates. In nearly all counting experiments on radioactive fluctuations made up to the present time, the detecting device subtended only a small solid angle at the source,[5] so that randomness in the direction of emission was superposed on the time distribution of the disintegrations. It has often been stated[6] that experiments in which the detecting device subtends a very small solid angle at the source cannot serve as a test of fluctuation theory. This is easily seen by considering an extreme and fantastic case,—a hypothetical source, emitting particles at uniform intervals. If it yields f particles in unit time, the probability that in time t n particles will pass through a very small solid angle ω is, by Poisson's formula, $(Aft)^n e^{-Aft}/n!$, where $A=\omega/4\pi$; but this is exactly what we would expect from a source subject to fluctuations, yielding an average of Af particles per unit time, in the solid angle ω. Similarly, if the efficiency, g, of the counter[7] in detecting a single particle is very small, the time-distribution of counts will follow Bateman's formula even though the counter receives the entire output of the above-mentioned hypothetical source.

However, we can prove that *if g and A are not very small compared with unity, a definite test of*

[4] Czuber, *Wahrscheinlichkeitsrechnung*, Vol. 1, p. 78.

[5] There are two exceptions. Kohlrausch and Schweidler (Physik. Zeits. **13**, 11 (1912)) attempted to utilize the entire emission, but Kohlrausch says the observations did not get beyond a qualitative stage. Curtiss (Bur. Standards J. Research **8**, 329 (1932)) used nearly the entire emission from a polonium source deposited on a thin foil between two counters.

[6] See for example, Kohlrausch, *Radioaktivität*, p. 789.

[7] Such an efficiency factor may arise in many ways. For example, in recording x-rays with a tube counter, only a part of the rays undergoes absorption or scattering in such a way that counts occur. The efficiency will of course be different for different kinds of rays or particles. (Section 6.) If several efficiency factors are active in sequence, we simply use an overall factor equal to their product; thus, if v is the probability that a photon will liberate an electron in the wall of a counter, and w the probability that this electron will produce a count, $g=vw$.

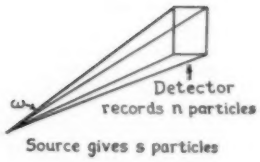

FIG. 2. In a typical case the source emits s particles in the interval (T_1, T_1+T_2); the detector records n of them.

the fluctuation theory can be made. For generality, we discuss a decaying source containing N atoms at time zero. The chance that the source will emit s particles or rays in the interval (T_1, T_1+T_2) is $W_s(T_1, T_1+T_2)$, given by Eq. (12); when this event happens, the chance (Fig. 2) that any one of the s particles will be recorded is gA, and the chance that n of them will be recorded is

$$C_n^s(gA)^n(1-gA)^{s-n}. \tag{14}$$

Therefore the chance of recording n particles in the interval T_2, however many are emitted by the source, is

$$\begin{aligned}&P_n(T_1, T_1+T_2)\\&=\sum_{s=n}^{N} W_s(T_1, T_1+T_2)C_n^s(gA)^n(1-gA)^{s-n}\\&=C_n^N e^{-N\lambda(T_1+T_2)}[1+e^{\lambda T_2}(e^{\lambda T_1}-1)]^N\\&\quad\times\left(\frac{gA}{1-gA}\right)^n\sum_{s=n}^{N}\frac{(N-n)!}{(N-s)!(s-n)!}x^s,\end{aligned} \tag{15}$$

where

$$x=\{(e^{\lambda T_2}-1)(1-gA)/[1+e^{\lambda T_2}(e^{\lambda T_1}-1)]\}.$$

The summation in Eq. (15) is $x^n(1+x)^{N-n}$, so that

$$\begin{aligned}&P_n(T_1, T_1+T_2)=C_n^N e^{-N\lambda(T_1+T_2)}(e^{\lambda T_2}-1)^n(gA)^n\\&\cdot[1+e^{\lambda T_2}(e^{\lambda T_1}-1)+(1-gA)(e^{\lambda T_2}-1)]^{N-n}.\end{aligned} \tag{16}$$

This is a generalization[8] of Eq. (12), and from it certain interesting facts emerge.

(1) *If N approaches infinity, but $N\lambda$ and n remain finite, then T_1 drops out of Eq.* (16) *and we have the Bateman formula for the probability of n counts in any interval of length T_2:*

$$P_n(T_2)=(N\lambda gA\,T_2)^n e^{-N\lambda gAT_2}/n! \tag{17}$$

(2) While the time-distributions of counts and of disintegrations are of the same form for a "constant" source, this is not true for short-lived sources.

(3) The mean number of counts in the interval T_2 is

$$\bar{n}=Ne^{-\lambda T_1}(1-e^{-\lambda T_2})gA, \tag{18a}$$

and the mean square deviation is

$$\begin{aligned}\overline{n^2}-\bar{n}^2&=Ne^{-\lambda T_1}(1-e^{-\lambda T_2})gA\\&\quad\times[1-e^{-\lambda T_1}(1-e^{-\lambda T_2})gA].\end{aligned} \tag{18b}$$

Many writers have used the value of the Lexian ratio, as a criterion for the random character of radioactive disintegration. For sources whose decay during a single experiment can be neglected, this ratio should be unity, whatever solid angle is subtended by the counting device. The case of a decaying source was discussed by Bortkiewicz[9] but he did not consider the influence of counter efficiency and solid angle. His discussion covers the frequency distribution of n-values in equal intervals of time, for a single source; that is, T_2 is constant, and T_1 increases by equal steps. For this case, the expected value of the Lexian ratio is greater than unity. On the other hand, the Lexian ratio given by Eqs. (18a) and (18b), is less than unity. It refers to the case in which we choose fixed values of T_1 and T_2, make experiments on a great number of sources, and consider the frequency-distribution of n values in the interval T_2. The complicated form of Eqs. (12) and (16) and of similar equations which follow, makes it inconvenient to use the Lexian ratio in studying decaying sources. On the other hand, $P_0(T_1, T_1+T_2)$ is simple in form, and if we determine by experiment the dependence of this function on T_1 or T_2, a comparison with theory can be made by a method employed by Bortkiewicz.[10] This is the most convenient procedure.

[8] Unfortunately Eq. (13) cannot be generalized to take account of g and A. It can be shown that when r *counts* have been observed, the probability that the interval between the rth and $(r+1)$th counts will exceed t is

$$[1-e^{-\lambda T_r gA(1-e^{-\lambda t})}]^{N-r},$$

where T_r is the time of occurrence of the rth count. Since this expression depends on T_r as well as r, the method of proof which led to (13) can no longer be invoked.

[9] Bortkiewicz, reference 2, p. 72.

[10] Reference 2, pp. 5–15.

5. Determination of fluctuations in disintegration when the distribution of counts is a known function

Suppose the functions $P_n(T_1, T_1+T_2)$ have been found by experiment. *When N is finite, and T_1 and T_2 are given fixed though arbitrary values, Eq. (15) constitutes a set of linear equations that can be solved for the functions W_s in terms of the functions P_n and the solution is unique. Thus experiments on the time-distribution of counts can give us a definite test of the probability theory of radioactive decay.*

To solve the Eq. (15) when T_1 and T_2 are given fixed values, we omit the arguments of the functions W_s and P_n, and write

$$v_s = W_s(1-gA)^s; \quad p_n = P_n\left(\frac{1-gA}{gA}\right)^n. \quad (19)$$

Then (15) assumes the simple form

$$\sum_{s=n}^{N} C_n^s v_s = p_n. \quad (20)$$

The work is done in Appendix 1 and the result is

$$W_j = (1-gA)^{-j}(-1)^j \sum_{i=j}^{N} C_j^i \left(\frac{1-gA}{gA}\right)^i (-1)^i P_i. \quad (21)$$

Since this result holds for every choice of T_1 and T_2, it is the general solution of our problem.

6. Distribution of counts due to gamma-rays and secondary beta-rays; counts due to two or more independent sources

Gamma-rays and secondary beta-rays are emitted very quickly after a disintegration, but this does not mean that their time-distribution is the same as that of disintegrations. Some transitions leave the daughter nucleus in the ground state and in general, a given gamma-ray is emitted only in a certain fraction of the disintegrations. The distribution of counts due to gamma-rays is found as follows. Let G_j be the probability of emission of a gamma-ray of frequency ν_j when a given atom disintegrates, and g_j the counter efficiency for this ray. The effective solid angle subtended at the source by the detector may also be different for the various rays,

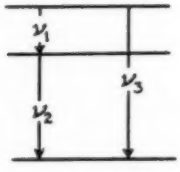

FIG. 3.

because of differences in their penetrating power. The value of this solid angle for the jth ray may be denoted by A_j. Then, the probability that a gamma-ray of this frequency will be emitted and will produce a count is $G_jA_jg_j$; and if the gamma-rays are mutually exclusive the total probability that the given disintegration will result in a count is $\sum G_jA_jg_j$. Replacing gA by this sum, the problem is treated exactly as in Section 4.

The situation is different if some of the gamma-rays are emitted in sequence without measurable time lag, which is often the case. To illustrate this, consider a hypothetical nucleus that emits only three gamma-rays, shown in the energy diagram (Fig. 3). Let the emission probabilities be G_1, G_2, G_3, where G_1 obviously equals G_2 and let the counter efficiencies be g_1, g_2, g_3. Then we can show that the probability a count will result from a given disintegration is $G_1(A_1g_1 + A_2g_2 - A_1A_2g_1g_2) + G_2A_3g_3$; this quantity replaces gA in the analysis of Section 4.

Let us now discuss the distribution of counts when two or more sources are superposed, or mixed. The superposition of any number of *independent* sources obeying Bateman's formula gives a resultant distribution obeying Bateman's formula; to be exact, if the average numbers of events caused by the individual sources in time t are x_1, x_2, etc., then the probability that all of them together will yield n events in time t is $(\sum x)^n e^{-\Sigma x}/n!$ The sources may be at different distances from the counter, may give rays of different absorbability, or may differ in any other respect; still the formula holds. The situation is quite different for decaying sources. By Eq. (16), the probability of n counts due to two such sources in the interval T_2 is

$$\sum_{j=0}^{n} P_j^{(1)} P_{n-j}^{(2)},$$

where the superscripts refer to the individual

sources. This is not of the form (16), unless the decay constants are equal and the products gA are the same for the two sources. Of course, when these quantities are equal we are dealing essentially with a single source.

C. Distribution of Counts in a Detector with Finite Recovery Time

Preceding sections dealt with the distribution of counts in a detector characterized by constant efficiency factors for the different classes of particles or rays which it receives, but having a negligible recovery time. Now we shall consider the distribution of counts in a detector whose efficiency drops to zero when a count occurs. We shall suppose (1) that it remains zero for a fixed time τ; and (2) that the counter then returns to a state of perfect efficiency.[11] We wish to find $P_n(0, t)$, the probability that n *counts* occur in the interval $(0, t)$. The general problem is complex, and we shall deal only with the case of a constant source.

7. Counts due to a constant source; counter not clogged at $t=0$

Let the counter receive f events per second, on the average. Then $P_0=e^{-ft}$. Up to the time $t=\tau$, P_2, P_3, etc., are zero, for if one count occurs anywhere in this interval, the counter is clogged for the remainder of the interval. Therefore, if $t\leqq\tau$:

$$P_1=1-P_0=1-e^{-ft}. \tag{22}$$

Now, let t be greater than or equal to τ, and consider the probability of just one count, occurring between s and $s+ds$. There are two cases, as shown at the bottom of Fig. 4A.

(1) If $s\leqq t-\tau$, the probability of one count in ds is (prob. of no event up to s)$\times$(prob. of 1 count in ds)$\times$(certainty that no further count occurs up to $s+\tau$, the counter being clogged)$\times$(prob. of no event from $s+\tau$ to t) $=e^{-fs}fds\,e^{-f(t-s-\tau)}$.

(2) If $s\geqq t-\tau$ we omit the last factor.

[11] In a tube or point counter, the recovery time is not constant, and the efficiency increases as recovery proceeds. These matters are discussed in Section 10, but our present purpose is to construct an approximate theory, applicable at low counting rates. When such complications are neglected, we need not retain the factors g and A in discussing a constant source, for we may confine our attention to the class of events which would be *counted* if the recovery time were zero; these events, of course, constitute a Bateman distribution.

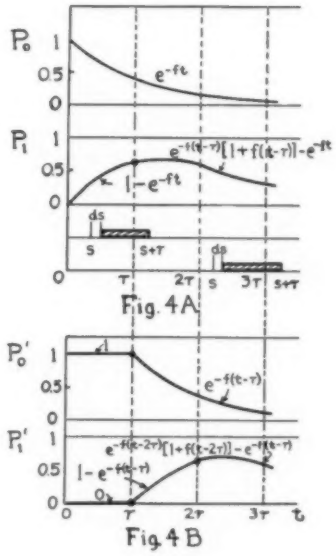

FIG. 4A. Probabilities of no counts and of one count in detector which is not clogged when $t=0$; and diagram illustrating derivation of Eq. (23).

B. Probabilities of no counts and of 1 count in detector which is clogged when $t=0$. The curves are drawn for the case $f=1/\tau$.

Therefore, if $t\geqq\tau$:

$$P_1=\int_0^{t-\tau}e^{-f(t-\tau)}fds+\int_{t-\tau}^{t}e^{-fs}fds$$
$$=e^{-f(t-\tau)}[1+f(t-\tau)]-e^{-ft}. \tag{23}$$

By similar methods P_n may be computed. The work is greatly shortened by the following facts: (1) $P_n=0$ if $t\leqq(n-1)\tau$. (2) In the range $(n-1)\tau$ to $n\tau$, P_{n+1} and all higher P's are zero, so in this range $P_n=1-P_1\cdots-P_{n-1}$. Thus we only need to compute P_n for the case $t\geqq n\tau$. In this range its analytic form does not change. The work is carried out in Appendix 2, and the result is

$$P_n(0, t)=F_n(t)-F_{n-1}(t), \tag{24}$$

where

$$F_n(t)=e^{-f(t-n\tau)}\{1+f(t-n\tau)+\cdots$$
$$+[f(t-n\tau)]^n/n!\}. \tag{25}$$

If we choose a value of t between $(s-1)\tau$ and $s\tau$,

$$\bar{n}=sF_s-\sum_{j=0}^{s-1}F_j. \tag{26}$$

8. Counts due to a constant source: count occurs at $t=0$

The above formulae are not quite what we want for the analysis of experimental data, where it is convenient to know the probability $P_n'(0, t)$ of n counts in the interval $(0, t)$ *after an initial count* at $t=0$. Up to time τ, $P_0'=1$ because the counter is clogged by the initial count. At time τ conditions are the same as at $t=0$ in the previous treatment, so that we may write

$$t \leqq n\tau : P_n'=0,$$

$$n\tau \leqq t \leqq (n+1)\tau : P_n'=1-P_1'\cdots-P'_{n-1},$$

$$t \geqq (n+1)\tau : P_n'=G_{n+1}-G_n, \quad n\neq 0;$$

$$P_0'=e^{-f(t-\tau)}; \quad (27)$$

where

$$G_n=e^{-f(t-n\tau)}\{1+f(t-n\tau)+\cdots + [f(t-n\tau)]^{n-1}/(n-1)!\}. \quad (28)$$

To obtain $\bar{n}$, suppose t lies between $s\tau$ and $(s+1)\tau$. Then

$$P_s'=1-\sum_0^{s-1} P_n'=1-e^{-f(t-\tau)}-\sum_1^{s-1} P_n',$$

where the terms in the summation are given by the last line of (27). Therefore

$$\bar{n}=\sum_1^{s-1} nP_n'+s-se^{-f(t-\tau)}-s\sum_1^{s-1} P_n'.$$

Now, $\sum_1^{s-1} P_n'$ is a telescoping series whose sum is G_s-G_1, and

$$\sum_1^{s-1} nP_n'=\sum_1^{s-1}[(n+1)G_{n+1}-nG_n]-\sum_1^{s-1} G_{n+1};$$

here the first sum telescopes, and is equal to sG_s-G_1, so that

$$\bar{n}=s-\sum_1^{s} G_n, \quad (29)$$

and by similar methods

$$\overline{n^2}=s^2-\sum_1^{s}(2n-1)G_n. \quad (30)$$

For practical purposes, we usually need only P_0', the fraction of the intervals between adjoining counts which are greater than t. We may obtain this fraction experimentally for various values of t and plot the results on semi-log paper. Beginning at abscissa τ, we have $\log_{10} P_0'=-0.4343f(t-\tau)$, so the experimental points should lie in the neighborhood of a straight line whose slope is $-0.4343f$. Thus *f can be determined without a knowledge of τ*, and the "goodness of fit" of the experimental points can be tested by several methods which need not be explained here.

9. General equations governing P_n

In setting up general equations for P_n, one naturally defines a function F_n, such that $F_n dt$ is the probability of a count in t to $t+dt$ when there have been n counts between 0 and t. Then we have

$$dP_n/dt=F_{n-1}P_{n-1}-F_nP_n, \quad (31)$$

the derivation being exactly like that of Eq. (2). However, the F's are discontinuous in analytical form, and study of their properties leads one to other equations which will now be obtained directly. Consider the probability of n counts in time $t+dt$. They may be realized in three ways:

(1) n in $(0, t-\tau)$; zero in $(t-\tau, t)$; zero in dt;
(2) $n-1$ in $(0, t-\tau)$; zero in $(t-\tau, t)$; 1 in dt;
(3) $n-1$ in $(0, t-\tau)$; 1 in $(t-\tau, t)$; zero in dt.

Let ${}_nP_s(t-\tau, t)$ be the probability of s counts in $(t-\tau, t)$, when n have preceded in the interval $(0, t-\tau)$. Then a little reflection will show that

$$P_n(0, t+dt)=P_n(0, t-\tau)\cdot {}_nP_0(t-\tau, t)(1-fdt) + P_{n-1}(0, t-\tau)\cdot {}_{n-1}P_0(t-\tau, t)fdt + P_{n-1}(0, t-\tau)\cdot {}_{n-1}P_1(t-\tau, t)(\text{unity}).$$

Separating the finite terms and the terms containing dt, we get

$$P_n(0, t)=P_n(0, t-\tau)\cdot {}_nP_0(t-\tau, t) + P_{n-1}(0, t-\tau)\cdot {}_{n-1}P_1(t-\tau, t); \quad (32)$$

$$dP_n(0, t)/dt=fP_{n-1}(0, t-\tau)\cdot {}_{n-1}P_0(t-\tau, t) - fP_n(0, t-\tau)\,{}_nP_0(t-\tau, t). \quad (33)$$

The evaluation of the functions ${}_nP_0$, etc., by integration is difficult, so Eqs. (32) and (33) are not very useful as a means of finding the functions P_n. On the contrary, they enable us to get the functions ${}_nP_0$, etc., from the values of P_n, given in Section 7.

10. Distribution of counts in counter with variable recovery time

Skinner[12] has obtained the distribution of recovery times in a counter exposed to a "constant" source. His results are based on Danforth's[13] studies of the way in which the counter voltage V varies when a discharge is initiated. Two classes of counts are considered. In the first, obtained at voltages close to the threshold, V drops with great rapidity to a value below the threshold, and recovers exponentially, with a time constant determined by the capacity of the counter and by the resistance in series with it. The recovery time depends on the time required for V to rise to the threshold. In the second type, encountered at higher voltages, V falls, recovers to the threshold value and remains close to the threshold for an indefinite period. The mechanism of the second type is not fully understood and we confine our remarks to the first type. Skinner adopts the reasonable view that there is space charge in the counter when effective ionization ceases, and that the voltage goes below the threshold because this charge is swept to the electrodes. How far V goes below the threshold depends on the voltage existing when the count occurs, and we must refer the reader to Skinner's paper for details. He tacitly assumes that the counter returns to complete efficiency when the voltage rises to the threshold. Of course this is not true, but it is a very desirable simplifying assumption which probably does not affect the results to any great extent at low counting rates.

Bearing in mind that our results depend on these assumptions, let us suppose that the probability of an ionizing particle in dt is fdt, f being constant. Skinner finds that when the counter is recording such a distribution of events, the probability of a recovery time between τ and $\tau+d\tau$ is

$$\omega(\tau)d\tau = f(d-1)^{f\,RC}(d-e^{\tau/RC})^{f\,RC-1}e^{\tau/RC}d\tau, \quad (34)$$

where RC is the time constant of the counter, and d is the maximum voltage drop divided by the difference between operating and threshold voltages; thus d is a constant which can be determined experimentally. The τ's have all sizes between zero and a certain maximum, $\tau_{\max}$, which is $RC\log_e d$.

The probability that no count occurs in an interval t, after a count whose recovery time is between τ and $\tau+d\tau$, is unity if $t \leqq \tau$, and $e^{-f(t-\tau)}$ if $t \geqq \tau$. Averaging this over all possible recovery times with the aid of Eq. (34), we get the over-all probability H_0, of no count in an interval t after any count whatsoever. If $t \geqq \tau_{\max}$, which is the only case of interest here,

$$H_0 = \int_0^{\tau_{\max}} \omega(\tau)e^{-f(t-\tau)}d\tau = e^{-f(t-\tau')}, \quad (35)$$

where

$$e^{f\tau'} = \int_0^{\tau_{\max}} \omega(\tau)e^{f\tau}d\tau.$$

This can be evaluated in series, but we only need to note that H_0 is of the same form as P_0, with τ' replacing τ. Therefore the introduction of a variable recovery time does not alter the form of the distribution of intervals; but we must remember that variation of efficiency with voltage was neglected.

D. Fluctuations in the Stock and the Disintegration of Daughter Substances

It was clearly recognized by Adams[14] that the emission from an entire radioactive series has fluctuations which do not obey the Bateman formula, even when the substances in the series are in equilibrium. He showed that if the interval of time for which probabilities are calculated is very short compared with the mean lives of some of the substances, and very long compared with those of others, the fluctuations can be computed approximately in terms of Bateman

[12] S. M. Skinner, Phys. Rev. **48**, 438 (1935). We are indebted to Dr. Skinner for placing his results at our disposal in advance of publication.

[13] Danforth, Phys. Rev. **46**, 1026 (1934). The authors have often made visual oscillographic observations which agree in a qualitative way with those of Danforth.

[14] N. I. Adams, Phys. Rev. **44**, 651 (1933).

functions. He did this for the alpha-particles of the thorium series, using an interval of 5 minutes which roughly satisfies the above requirements. Adams' analysis refers to the equilibrium case, and he did not consider the influence of fluctuations in the stocks of the various substances. We shall now develop methods for dealing with all cases rigorously.

11. Theory of stock fluctuations

At time zero, let a source consist of N atoms of the parent substance of a radioactive series whose members are labeled A, B, $\cdots J$. The decay constants will be denoted by λ_A, λ_B, $\cdots$, where the subscript A belongs to substance A, and so on. The results of simple disintegration theory give us the probabilities a, b, $\cdots j$ that one of these N atoms, chosen at random, will be of type A, B, $\cdots J$, respectively, at a later time t. The probability that the stocks of A-atoms, etc., will be n_A, $n_B\cdots$ at time t is

$$S(n_A, n_B\cdots;t)=\frac{N!}{n_A!\,n_B!\cdots n_J!}a^{n_A}b^{n_B}\cdots j^{n_J}, \quad (36)$$

and the probability of a stock n of B-atoms, without regard to the nature of the others, is simply

$$S_n(t)=C_n{}^N b^n(1-b)^{N-n}. \quad (37)$$

In a series composed of a parent A, a daughter B, and an end-product C, with only the parent present at the start, these probabilities are

$$a=e^{-\lambda_A t};\quad b=\frac{\lambda_A}{\lambda_B-\lambda_A}(e^{-\lambda_A t}-e^{-\lambda_B t});$$

$$c=\frac{\lambda_B}{\lambda_B-\lambda_A}(1-e^{-\lambda_A t})-\frac{\lambda_A}{\lambda_B-\lambda_A}(1-e^{-\lambda_B t}), \quad (38)$$

which will be useful farther on. To treat the important case in which the parent A is very long-lived, and is the only substance present initially, we let N approach infinity and λ_A approach zero, keeping $N\lambda_A$ equal to a constant f. Then a approaches 1 and b approaches 0, but

$$Nb\to(f/\lambda_B)(1-e^{-\lambda_B t})\equiv x, \quad (39)$$

and

$$S_n(t)=x^n e^{-x}/n!, \quad (40)$$

x being the "expected" number of B-atoms at time t. Thus, when t approaches infinity, so that equilibrium would exist according to simple disintegration theory, the stock fluctuations of B-type atoms are given by $S_r=(f/\lambda_B)^n e^{-f/\lambda_B}/n!$ Similarly, the stock-fluctuations of all substances except the parent and the end-product obey the formula

$$S_n=x^n e^{-x}/n!, \quad (41)$$

where x is the "expected" value of the stock.[15]

12. Recurrence equations governing stock fluctuations

A differential equation governing the functions $S_n(t)$ will now be obtained. Consider a given kind of atom, say the kind B, and let $f_n dt$ and $p_n dt$ be the probabilities that an atom of this kind will be formed, or will decay, respectively, in the interval dt at t, if the stock of B-atoms is n. (The function f_n depends on this stock, because the probable stock of the parent A depends on n, among other variables.) Now let us consider the value of $S_n(t+dt)$. There are three ways[16] in which a stock of n B-atoms can be realized at time $t+dt$.

(1) stock $=n-1$ at t; 1 is formed and 0 decay in dt;

(2) stock $=n$ at t; 0 are formed and 0 decay in dt;

(3) stock $=n+1$ at t; 0 are formed and 1 decays in dt.

The probability sought is the sum of three probabilities:

$$S_n(t+dt)=S_{n-1}(t)f_{n-1}dt+S_n(t)[(1-f_n dt)(1-p_n dt)]+S_{n+1}(t)[(1-f_{n+1}dt)p_{n+1}dt]. \quad (42)$$

It follows that

$$dS_n/dt=f_{n-1}S_{n-1}-(f_n+p_n)S_n+p_{n+1}S_{n+1}. \quad (43)$$

When $n=0$, the first term drops out. Because of the presence of S_{n+1}, this is not a differential

[15] When t approaches infinity, the amount of the parent approaches a value which depends on the value assigned to M. Also, the amount of the end product is infinite and fluctuations of these substances have no meaning and no interest.

[16] Cases in which several B-atoms are formed and several decay in dt can be neglected because they give rise to infinitesimals of higher order than those which must be retained.

equation which we can integrate to get S_n when we know S_{n-1}. It is really a recurrence relation for the *seriatim* computation of S_1, S_2, etc., when we know S_0.

13. Fluctuations in the disintegration of a daughter atom and of a series

When a parent substance and its descendants are present in the same source the fluctuations in the total emission do not follow the simple formulas derived in Sections 3 to 6. The problems which arise are very numerous, and when the decay of the parent must be taken into account the algebra is complex. To illustrate as comprehensively as possible the methods of solving such problems, we limit the discussion to a hypothetical three-member series composed of parent, daughter and end-product substances, called A, B, and C, respectively. It is convenient to refer to the disintegrations of parent and daughter as A-emissions and B-emissions, respectively.

The new feature that arises is this: if the stocks of A- and B-atoms are n_A and n_B at time zero, B-emissions in the interval $(0, t)$ may arise in two ways. (1) Some of the B-atoms present at time zero may disintegrate, the fluctuations being given by Eq. (10); and (2) some A-atoms may undergo two disintegrations, thus contributing to the B-emissions. The problem is to find the fluctuations in the B-emission, and also the fluctuations in the total emission. We shall solve a simple problem in which the decay of the parent stock must be allowed for, and shall then deal with cases in which the parent is very long-lived.

Fluctuations when all atoms are initially of kind A. Let there be N A-atoms initially. The probability of n B-emissions in the interval $(0, t)$ is the probability that n of the A-atoms change into C-atoms in time t. This is

$$W_{n,\,B}(0,t)=C_n^N c^n(1-c)^{N-n}, \qquad (44)$$

where c is the probability that any given atom will be of kind C; c is given by Eqs. (38). To get the probability of n emissions from the whole source, we consider a typical case in which k A-atoms go into the B-state and remain there until time t at least, and l A-atoms go into the C-state; it is understood that $k+2l=n$. We sum the probabilities of all such cases. If n is even, the result is

$$W_{n,\,A+B}(0,t)=\sum_k \frac{N!}{(N-k-(n-k)/2)!\,k!\,((n-k)/2)!} \times a^{N-k-\frac{1}{2}(n-k)}b^k c^{\frac{1}{2}(n-k)}, \qquad (45)$$

where $k=0, 2, \cdots n$. If n is odd the summand is the same but $k=1, 3, \cdots n$.

To obtain the fluctuations in B-emission when the parent is very long-lived, and there are no B-atoms initially, we allow N to approach infinity and λ_A to approach zero in such a way that $N\lambda_A=f$. Then (44) is replaced by

$$W_{n,\,B}=x^n e^{-x}/n!; \quad x=ft-(f/\lambda_B)(1-e^{-\lambda_B t}); \qquad (46)$$

x is simply the number of atoms which should pass into the C-state in time t according to disintegration theory.

Fluctuations of emission by daughter in equilibrium with long-lived parent. Suppose that the daughter B should be in equilibrium with its parent, but that owing to fluctuations the daughter stock at time zero is n, a value which may or may not be the equilibrium one. Suppose k of these B-atoms emit in time t, and that l of the parent atoms change into C-atoms; and let $k+l=s$. By using (40) and (46), and replacing λ_B by λ for simplicity, the probability of this composite event is

$$S_n C_k^n (e^{\lambda t}-1)^k e^{-n\lambda t} x^l e^{-x}/l! \qquad (47)$$

To get the total probability of s B-emissions we must sum (47) over all allowable values of n and k. Obviously, n must be at least equal to k so n runs from k to ∞ and k runs from 0 to s. Replacing l by $s-k$, and n by $m+k$, we have, after some reductions,

$$W_{s,\,B}=\frac{e^{-f/\lambda}e^{-x}}{s!}\sum_{k=0}^{s} C_k^s \left(\frac{f}{\lambda}\right)^k \cdot\left(\frac{1-e^{-\lambda t}}{x}\right)^k x^s \sum_{m=0}^{\infty}\frac{(fe^{-\lambda t}/\lambda)^m}{m!}$$

$$=(ft)^s e^{-ft}/s! \qquad (48)$$

Thus *the emission of the daughter atom fluctuates in the same way as that of the long-lived parent.*[17]

[17] The simplicity of this result is surprising, when we consider that Eq. (48) results from stock-variations and

Fluctuations of total emission under equilibrium conditions. It must not be supposed that the total emission of parent and daughter follows the Bateman formula. In Section 6 we remarked that the superposition of *independent* sources fluctuating according to the Bateman formula gives rise to a distribution of events obeying that formula, but in the present instance the sources are not independent; fluctuations in the emission by the parent have a direct influence on the subsequent emission of the daughter. To illustrate this, if for a time the parent disintegrates at a rate higher than the average the stock of the daughter will be raised and it will emit at a rate higher than average. Before working out the general case, let us clarify our ideas by getting W_0, the chance of no emission by both parent and daughter in time t. Now, the probability that (1) there are n B-atoms at time zero; (2) no A-atoms disintegrate in time t; and (3) no B-atoms disintegrate in time t, is $S_n e^{-ft} e^{-n\lambda t}$, and averaging over n,

$$W_0 = e^{-ft-(f/\lambda)(1-e^{-\lambda t})}. \tag{49}$$

When λt approaches zero, this function behaves like e^{-2ft}, and when λt is large compared with unity, it behaves like e^{-ft}. It is instructive to consider the limiting case in which the life of the daughter is very short compared with the average interval between disintegrations of parent atoms. Then nearly all the events occur in close pairs. The average spacing of the events composing a pair is of the order $1/\lambda$, and the longer intervals between pairs are distributed approximately according to the law e^{-ft}, being governed mainly by disintegrations of the parent atoms.

Now we shall obtain the probability of s emissions by both parent and daughter. Consider the case in which there are n B-atoms at time 0, and in time t, (1) k B-atoms disintegrate; (2) l A-atoms undergo one disintegration each; (3) m A-atoms undergo two disintegrations each. We first require the probability that both the events (2) and (3) occur. Consider a source containing N A-atoms with finite decay constant, and pass to the limit $N\to\infty$, $\lambda_A\to 0$, $N\lambda_A = f$. The probability that both (2) and (3) occur is

$$[N!/(N-l-m)\,!\,l\,!\,m\,!]a^{N-l-m}b^l c^m,$$

where a, b, and c are given by Eqs. (38), and in the limit this becomes

$$e^{-ft}y^l x^m/l\,!\,m\,!, \tag{50}$$

where y is an abbreviation for $(f/\lambda)(1-e^{-\lambda t})$, $x = ft - y$, and λ is written in place of λ_B. Finally, replacing l by $s-k-2m$, the total probability of s disintegrations by both parent and daughter is, when s is even,

$$W_{s,\,A+B} = \sum_{m=0}^{s/2} \sum_{k=0}^{s-2m} \sum_{n=k}^{\infty} S_n C_k^{\,n} (e^{\lambda t}-1)^k e^{-n\lambda t} \times e^{-ft} y^{s-k-2m} x^m/(s-k-2m)\,!\,m\,!.$$

Putting $n = k+j$ for convenience, the result is this:

$$W_{s,\,A+B} = (2y)^s e^{-ft-y} \sum_{m=0}^{s/2} (x/4y^2)^m/(s-2m)\,!\,m\,!. \tag{51}$$

When s is odd, m runs from 0 to $(s-1)/2$.

The results of this section can be tested in a very direct way. Suppose we prepare a source of an alpha-raying substance whose daughter is an alpha-rayer of short life, adjusting the strength so that both parent and daughter give only a few particles per hour when equilibrium is attained. Using a tube electrometer, the particles emitted by these two substances can be distinguished from those due to their descendants, and also from each other. Thus we can obtain all the data needed for tests of Eqs. (48) and (51), and also of Eq. (10).

In conclusion, the formulae derived in this paper are sufficient in scope to facilitate the solution of a wide variety of fluctuation problems. To keep the paper within bounds, we have omitted applications to experimental data and have made no mention of fluctuation-velocities, an important though neglected subject. It is a pleasure to acknowledge our indebtedness to Professor Forrest Western, who collaborated in experimental work which suggested the writing of this paper.

emission-fluctuations of the kind considered above. The reasoning has been checked by passing to the limit after setting up the problem for the case of a parent having a finite decay constant.

Appendix 1. Solution of Eq. (20)

The problem is to find the quantities v_s from the equations

$$\sum_{s=n}^{N} C_n^s v_s = p_n; \quad n=0, 1, \cdots N. \tag{20}$$

If we differentiate the identity

$$(a+b)^s = \sum_{n=0}^{s} C_n^s a^{s-n} b^n$$

with respect to b, j times, we get

$$s(s-1)\cdots(s-j+1)(a+b)^{s-j} = \sum_{n=j}^{s} C_n^s a^{s-n} n(n-1)\cdots(n-j+1)b^{n-j}.$$

Putting $a=1$, and $b=-1$, we find that

$$\sum_{n=j}^{s} C_n^s C_j^n (-1)^{n-j} = \delta_{sj},$$

where δ_{sj} is 0 if $s \neq j$ and 1 if $s=j$. This suggests that to get v_j, we should multiply the typical equation in (20) by $C_j^n(-1)^{n-j}$. (Since C_j^n has no meaning if n is less than j, we begin, of course, with the jth equation.) Doing this and adding the results we have

$$\sum_{n=j}^{N}\sum_{s=n}^{N} C_n^s C_j^n (-1)^{n-j} v_s = \sum_{n=j}^{N} C_j^n (-1)^{n-j} p_n.$$

Interchanging the order of summation on the left, we have

$$\sum_{s=j}^{N} v_s \sum_{n=j}^{s} C_n^s C_j^n (-1)^{n-j}.$$

By the identity derived above, the coefficient of v_s vanishes unless $s=j$, and we have

$$v_j = \sum_{n=j}^{N} C_j^n (-1)^{n-j} p_n.$$

Appendix 2. Counts in a Detector with Constant Recovery Time, τ.

Let the counter receive, on the average, f events per second, distributed according to the Bateman formula. We wish to obtain P_n, the probability that n counts occur in the interval $(0, t)$. The counter is not clogged at $t=0$; and in accord with Section 7 we are interested only in the case where $t \geqq n\tau$. We shall call a period during which the counter is clogged an interlude. Two cases must be considered, as shown in Fig. 5. In Case 1, the nth interlude ends before the instant t, and in Case 2, it extends beyond this instant.

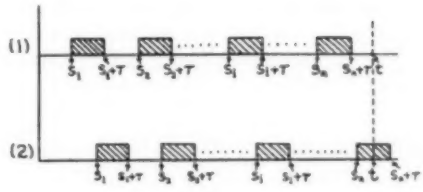

Fig. 5.

We begin by writing the probability that the n counts will occur, respectively, in the intervals (s_1, s_1+ds_1); (s_2, s_2+ds_2), $\cdots$, (s_n, s_n+ds_n), where it is understood, of course, that s_2 is greater than or equal to $s_1+\tau$, and so forth. In Case 1, this probability is $e^{-fs_1}fds_1 e^{-f(s_2-s_1-\tau)}fds_2 \cdots e^{-f(s_n-s_{n-1}-\tau)}fds_n e^{-f(t-s_n-\tau)} = e^{-f(t-n\tau)}f^n ds_1 \cdots ds_n$. This must be integrated for all possible positions of the counts that can occur in Case 1, ranging from the case in which all the interludes are closely packed at the left in Fig. 5, upper diagram, to that in which they are closely packed at the right, with the end of the nth interlude at the instant t. For example, the limits for s_1 are 0 and $s_2-\tau$; for s_{n-1} they are $(n-2)\tau$ and $s_n-\tau$; and for s_n they are $(n-1)\tau$ and $t-\tau$.

Similarly, in Case 2, the probability of having counts in ds_1 at s_1, etc., is

$$e^{f(n-1)\tau}e^{-fs_n}f^n ds_1 \cdots ds_n,$$

and the limits are the same as before, except for s_n, whose limits are now $t-\tau$ to t. Therefore

$$P_n = e^{-f(t-n\tau)}\cdot\int_{(n-1)\tau}^{t-\tau}\int_{(n-2)\tau}^{s_n-\tau}\cdots\int_{\tau}^{s_2-\tau}\int_{0}^{s_2-\tau} f^n ds_1 \cdots ds_n + e^{f(n-1)\tau}\int_{t-\tau}^{t}\int_{(n-2)\tau}^{s_n-\tau}\cdots\int_{\tau}^{s_3-\tau}\int_{0}^{s_2-\tau} e^{-fs_n}f^n ds_1 \cdots ds_n.$$

The substitution $x_i = f[s_i-(i-1)\tau]$ makes it simple to carry out the integration over $s_1, s_2 \cdots s_{n-1}$, and the result is $f^{n-1}[s_n-(n-1)\tau]^{n-1}/(n-1)!$, so finally

$$P_n = e^{-f(t-n\tau)}f^n\int_{(n-1)\tau}^{t-\tau}[s_n-(n-1)\tau]^{n-1}/(n-1)!\cdot ds_n + e^{f(n-1)\tau}f^n\int_{t-\tau}^{t} e^{-fs_n}[s_n-(n-1)\tau]^{n-1}/(n-1)!ds_n$$
$$= e^{-f(t-n\tau)}[1+f(t-n\tau)+\cdots+f^n(t-n\tau)^n/n!] - e^{-f(t-(n-1)\tau)}[1+f(t-(n-1)\tau)+\cdots + f^{n-1}(t-(n-1)\tau)^{n-1}/(n-1)!],$$

which is Eq. (24).

The Radiations Emitted from Artificially Produced Radioactive Substances

I. The Upper Limits and Shapes of the β-Ray Spectra from Several Elements*

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The β-ray spectra of N^{13}, F^{17}, Na^{24}, Si^{31}, P^{32}, Cl, A^{41} and K^{42} have been investigated by measuring the curvatures of the tracks due to the β-rays in a cloud chamber traversed by a known magnetic field. By allowing the tracks to be formed in hydrogen the scattering of the tracks is so reduced that the distribution curves are felt completely to represent the true distributions for momenta greater than $1000H\rho$. It is found that the shapes of these curves are in very good accord with the Konopinski-Uhlenbeck modification of the Fermi theory for the first five elements mentioned above (two positron emitters and three electron emitters). The spectra of the last three elements above can be resolved into two components, each of which has a shape which fits the theory. In the case of N^{13} an upper limit has been found from fitting the data with a theoretical curve at 1.45 MV. The K–U theory always indicates a higher upper limit than is found by inspection of the data, because of the high order of contact with the momentum axis which it demands. The upper limit of N^{13} can be calculated from known reaction energies and should be 1.5 MV. The excellent agreement of these two values of the upper limits is regarded as suggesting that the high K–U limits represent the true energy changes in a β-disintegration.

SINCE the discovery by Curie and Joliot[1] that several elements could be made radioactive by bombardment with α-rays others[2, 3, 4] have found that the same could be accomplished by the use of protons, deuterons or neutrons as the primary agents. These artificial elements are isotopes of known stable elements and are in general β-ray emitters. (We shall use the term "β-ray" to embrace both positrons and electrons.) The opportunity to study the very perplexing problem of β-emission is somewhat more favorable for these light elements, and for the following reasons: There are no subsequent products which may confuse the data; the opportunity is available to obtain independent values of the energy changes in β-emission by means of other nuclear reactions; the β-rays arise from light nuclei, which are presumably simpler; finally, it is very easy to prepare a great number of these bodies of such activities that one can do more careful work with them than is feasible with any but a few of the naturally radioactive elements.

The present work began simply as an attempt to arrive at values for the energy changes in β-emission, by measuring the radii of curvature of the tracks due to the β-rays from various deuteron-activated bodies in a cloud chamber traversed by a known magnetic field. It is generally agreed that the "upper limit" of a continuous β-ray distribution corresponds to the energy change of the reaction. This upper limit is usually practically defined as corresponding to the energy of the fastest track observed. Rigorously it must be an energy such that no track will ever be found with an energy exceeding it no matter how many tracks are measured. Naturally such a limit can be found experimentally only by extrapolation. If the distribution curve approaches the axis of momenta very steeply then it will be possible to set a value to the upper limit, by observing the fastest tracks, which may be very close to the true limit. However, if the distribution curve approaches the axis very slowly then such a practice will lead to low values of the limit. It seems to be more satisfactory to fit a theoretical curve to the data (in the form of a distribution histogram) and take the upper limit from it by extrapolation. Until recently it has been impossible to attempt this because of the absence of a suitable theory,

* Presented at the Berkeley meeting of the American Physical Society. The data given in this paper have all been recalculated and supersede anything given in the abstract of that meeting.

[1] I. Curie and F. Joliot, Comptes rendus **198**, 254 (1934); Nature **133**, 201 (1934).

[2] J. D. Cockcroft, C. W. Gilbert and E. T. S. Walton, Nature **133**, 328 (1934).

[3] M. C. Henderson, M. S. Livingston and E. O. Lawrence, Phys. Rev. **45**, 429 (1935).

[4] E. Fermi, Ricerca scientifica (1) **5**, 283 (1934).

but now a theory due to Fermi[5] together with a modification due to Konopinski and Uhlenbeck[6] is available. In getting an upper limit in this way, by fitting a theoretical curve to the data and extrapolating, it is important that the shape of the curve is not in any way falsified by experimental conditions.

Last spring we examined the Fermi theory to see if it predicted the shape of the distributions we were getting and reported favorably on the agreement between the two at the Washington meeting of the American Physical Society in April, with the reservation that we did not feel that our experiments were good enough satisfactorily to test a theory. At that time we were using oxygen as the gas in the chamber as is usual in β-ray work. In measuring the curvatures of the tracks we had adopted the rule that all tracks with visible deflections in them were to be rejected. That this was distorting the shape of the distribution curves we knew because we were being forced to discard many more of the low energy tracks than the high energy ones. This distortion can be reduced to a very great extent by photographing the β-tracks in hydrogen instead of oxygen. The scattering is thus reduced by a factor of 64 so that what little scattering remains is probably due to the heavier nuclei in the vapor.

We found with the hydrogen filled chamber that the distribution curves were more skew than they had appeared with the oxygen filled chamber. This is not surprising: our criterion of selection had been forcing us to discard as unmeasurable a large number of low energy tracks. The number discarded increased as the energy of the track decreased. The apparent concordance between our early data and the Fermi theory was entirely traceable to this because the Fermi distribution function is very nearly symmetrical, so that when the number of low energy tracks measured was increased this apparent symmetry in the experimental distributions was lost. At the suggestion of Dr. Arnold Nordsieck we attempted to fit our new data to the then unpublished modification of the Fermi theory due to Konopinski and Uhlenbeck. The K–U theory has the here desirable feature of asymmetry and was found to fit the experimental data very well. This symmetry carries with it, however, a high order of contact of the distribution curve with the momentum axis, which leads to an upper limit greater than the last measured track in most cases. Because of this we hesitated to attribute any physical significance to these higher upper limits. Recently, however, the reaction energy pertaining to a β-ray change has been established in one case (N^{13}, see later) independently of any β-ray measurements. This energy change is in excellent agreement with that indicated by fitting a K–U curve to the data on the β-ray spectrum.

We have reported on the excellent way in which the K–U theory accounts for the shape of the curves in a Letter to the Editor of this journal[7] and also in the nuclear symposium at the Los Angeles meeting of the American Physical Society, without urging the acceptance of the higher upper limits indicated by it.

In this paper we shall present distribution curves of the β-rays emitted from deuteron-activated carbon, oxygen, sodium, silicon, phosphorus, chlorine, argon and potassium. With them the K–U theory will be tested insofar as the shape of a distribution curve is concerned. From fitted theoretical curves we shall deduce values for the various endpoints, which we feel little hesitancy in offering as representing the energy change in the β-emission.

Experimental Details

With the magnetic resonance accelerator of Lawrence and Livingston[8] it has been possible to stimulate radioactivity in elements as heavy as platinum[9] using an intense deuteron beam of as much as 15 microamperes at as high as 5.3 MV. Depending upon the nature of the radioactive substance it is desired to create, a suitable target is bombarded in a vacuum or in air (the deuteron beam emerges through a thin window into a "target chamber") for a time sufficient to give

[5] E. Fermi, Zeits. f. Physik **88**, 161 (1934).
[6] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 7 (1935).
[7] F. N. D. Kurie, J. R. Richardson and H. C. Paxton, Phys. Rev. **48**, 167 (1935).
[8] E. O. Lawrence and M. S. Livingston, Phys. Rev. **45**, 608 (1935).
[9] J. M. Cork and E. O. Lawrence, Am. Phys. Soc., Berkeley Meeting, Phys. Rev. **49**, 205 (1936).

a convenient activity. In many cases it is desirable to protect the target from contamination in the target chamber by covering it with a thin foil, usually platinum. A great variety of targets have been used. The usual one consists of a fused salt of the element to be studied, the other component of the salt being generally one with either a very short half-life or with a low activity. Thus for studying radio-sodium it is convenient to use NaF, since the half-life of the fluorine activity (10 sec.) is much shorter than that of sodium (15 hours). Such a target is always "aged" for a sufficient time to allow the activity in the other component to decay to an insignificant magnitude. Similarly radio-chlorine may be examined in the form of $PbCl_2$ since the heavy element lead is activated to a much less degree if at all. Where possible the pure element is used as a target. Even gases may be bombarded by flooding the target chamber with them and collecting the radioactive elements by recoil on a piece of platinum foil.

The activated target is then put in one or the other of two holders mounted near the periphery of a Wilson chamber. The first and more convenient of these holders consists of a brass well set into the roof of the chamber with a grid, also of brass, covered by a thin mica window cemented on with gelatin. We have used this container for most of our work, because the cloud chamber can be kept clean and uncontaminated and sources can be dropped into it and photography begun immediately if desired. We have felt that there were objections to this holder for the following reasons: The mica window is so large that no definite solid angle for the β-rays is defined, the solid angle depending on the radius of curvature of the track and hence on its energy. (From a comparison of the shape of the distribution curves obtained with this holder and with the other (see later) which limits the solid angle more uniformly for all energies the conclusion has been reached that this effect is not serious.) The mica window had a thickness of about 8 mg/cm² and so caused a retardation of at most 60–80$H\rho$. The brass grid itself is only about 800 mg/cm² thick and so will allow electrons of 2 MV and over to penetrate it. This causes no trouble except for high energy spectra, such as the spectrum of chlorine, and here we do not rely on the data obtained with this holder alone.

The second holder was designed to eliminate these objections. It consists of a half cylinder of aluminum with an equatorial slot which confines the β-rays to the central plane of the cloud chamber. The walls of the aluminum slit are over 1500 mg/cm² thick and so will stop all β-rays of energy below 3 MV. It is made in the form of a half cylinder so that there will be no heavy backing to cause undue reflection of the electrons. This holder is something of a nuisance to use since the cloud chamber has to be taken down to insert a source and considerable time elapses before photography can begin. In general therefore we use the first holder for we have found that its shortcomings are not important: their influence on the shape of the distribution curve is only felt at the very low energies where all data are doubtful; the upper limit obtained is apparently independent of the holder. We shall mention as we consider the separate elements with what holder the data have been taken.

The cloud chamber is the same one mentioned by one of the authors previously.[10] It is 2 cm deep and 16 cm in diameter. For all the work presented in this paper it was filled with tank hydrogen and operated at about 100 cm pressure. The vapor was isopropyl alcohol. The tracks were illuminated by a shorted carbon arc and photographed with a single Sept camera equipped with an *f*2 Schneider lens of 50 mm focal length.

Above and below the cloud chamber are two coils of the following approximate dimensions: I.D. 8.36 inches, O.D. 9.93 inches, height of each coil 2.5 inches, separation of the two coils 2.56 inches. The coils are wound on a brass form, each carrying 829 turns of No. 18 wire. The two coils are connected in parallel and have a constant of 33.5 gauss per ampere. This figure has been obtained by comparing the deflections of a ballistic galvanometer when a standard search coil whose area-turns are known to less than 0.2 percent "by construction" was placed in the field of the chamber coils and when it was placed in the field of a standard solenoid whose constants were also known to less than 0.1 percent "by construction." All the values have been

[10] F. N. D. Kurie, Phys. Rev. **47**, 97 (1935).

checked against a standard mutual inductance of 50 millihenrys and agree to 0.3 percent. We regard our field as being accurately known to one percent at the center of the chamber. The ammeter by which the current is read during the photographing of a series of expansions has been calibrated from time to time against a standard cell and standard 0.1-ohm resistance and has been found to maintain its calibration within the accuracy to which it can be read (about one percent). The coils are closer together than the Helmholz position but over the region where tracks are formed the field is uniform to less than four percent. This would mean a maximum distortion of the measured curvature of a track of two percent if the field fell off linearly. Actually it falls off much less rapidly until the periphery of the chamber. Hence this effect can account for an error of approximately one percent in the radii of the tracks.

The time which the coils require to reach their maximum current has been estimated with a cathode-ray oscilloscope at between 1/120 and 1/60 sec. Since the coils are turned on more than a second before the chamber expands the field has ample time to reach maximum strength before the chamber is photographed.

On 110 volts d.c. the coils produce a field of about 500 gauss. They may also be used up to 220 volts giving a field of 1000 gauss. This range of fields covers β-rays from $1000H\rho$ to around $25{,}000H\rho$ (0.1–7 MV) with curvatures of an easily measurable magnitude. In order to prevent the coils from overheating the current is passed through them for only about two seconds during the expansion of the cloud chamber. At the high field the heating of the coils is sufficient to require the chamber to be water cooled in order that tracks be undisturbed. In general, however, the 500 gauss field is sufficient to cope with the β- and γ-rays encountered, at which field there is no thermal disturbance at all.

The film after development is replaced in the camera and reprojected on a screen placed at the same optical distance from the camera as the cloud chamber was when the pictures were originally taken. The tracks are measured by comparing them with a set of circles engraved on a sheet of celluloid. The circles have radii from 1 to 29 cm drawn at intervals of 0.5 cm. The error in measuring a curvature is less than five percent in the region of the upper limits. However, since it is our practice to plot the data in class intervals of either 500 or $1000H\rho$ this error does not appreciably alter the shape of the distribution curves. The use of hydrogen in the chamber so diminishes the scattering of the tracks that except for the very slowest (less than $1000H\rho$), which are in general not measured, few deflections are noticed in the tracks. Indeed it is an exceptional track which has a discernible deflection. The tracks in hydrogen are resolved into individual droplets; this makes them easier to measure than the broader tracks in oxygen, for one can fit the circles on the celluloid scale to the reprojected track by trying them for a bisection of the images of the droplets. This seems to us to be a more precise method of circle fitting than can be achieved with the oxygen tracks. Despite the very large reduction in scattering experienced with hydrogen occasional tracks which have had their curvatures falsified by small deflections will find their way into the final data, and this must be borne in mind in evaluating such data. Probably this effect is no worse than the everpresent chance in a magnetic spectrograph that a particle which has had many reflections from slits will be counted when a portion of the distribution curve which contains few particles is being examined.

The tracks are plotted as a histogram showing the number of tracks in a given $H\rho$ interval or in a given radius interval (either 0.5 cm or 1 cm depending on the number of tracks available). The latter is done in some cases rather than to plot against $H\rho$ directly to obviate systematic irregularities which may be introduced into the curve. Thus at 540 gauss the interval 1300–$1400H\rho$ will include only tracks of 25 cm radius while 1200–$1300H\rho$ will take in tracks of 23 and 24 cm radius and likewise 1400–$1500H\rho$ will contain 26 and 27 cm radius tracks. There will consequently always be a deficiency of tracks in the interval 1300–$1400H\rho$. For some field values this artificial irregularity does not exist, but when it does we have adopted the practice mentioned above and merely mark on the graph the appropriate $H\rho$ values. The relation between the $H\rho$ of a given track and its energy in MV is given by the following formula, which is valid

for all values of the energy:

$$H\rho = (10^4/3)(E(E+1.02))^{\frac{1}{2}}.$$

The number 1.02 is simply twice the rest mass of an electron expressed in millions of volts.

The histogram giving the frequency distribution in $H\rho$ may be transferred into a distribution in energy by dividing point by point each ordinate by the velocity of the electron.

Because of the difficulty of measuring small radii of curvature together with the increased number of rejections due to the augmented frequency of deflections in the tracks of low energy β-particles the portion of the histogram referring to them is in general stunted. With hydrogen and careful measuring this extends up to about $1000H\rho$, with oxygen it is apparent up to $4000H\rho$ and beyond. If the body being studied emits γ-rays along with its electronic radiations secondary electrons will be mixed with the disintegration β-particles, and will, if the γ-ray energy is greater than the β-ray upper limit, add a tail to the distribution which will again lead to a false value of the upper limit. These difficulties must be kept in mind in dealing with cloud chamber data.

We shall see that on the basis of any data on β-ray emission which we now possess the Konopinski-Uhlenbeck modification of the Fermi theory completely describes the process of the emission of a β-ray so that we may assign upper limits to the spectra by fitting to them a K–U curve.

Application of Theory to the Experimental Data

E. Fermi has given a theory of β-decay in which the emission of an electron by a nucleus is imagined as being occasioned by the transition of a nuclear neutron into a proton with the birth of an electron-neutrino pair. These two then escape from the nucleus with a constant total energy and the energy distribution of the electrons merely expresses the manner in which this total energy is shared between electron and neutrino. Theoretically this distribution depends upon the choice of interaction energy between the electron-neutrino field and the heavy particle. This interaction "Ansatz" is rather arbitrary and Konopinski and Uhlenbeck have shown[11] that several forms of the distribution function can be written down. They denote them by referring to the order of the derivatives of the electron and neutrino wave functions which enter into the coupling energy. Thus the (0,0) distribution is Fermi's original form; (0,1) is the form proposed by Konopinski and Uhlenbeck in their first paper. The general shape of the curves which we have experimentally found leads us to reject all the other forms which these authors have proposed, indeed, we shall see that only the (0,1) form requires consideration.

Eq. (40) of Fermi's paper gives the probability that an electron of momentum $mc\eta(\eta = H\rho/1700)$ in $d\eta$ be emitted. On integrating this from $\eta = 0$ to $\eta = \eta_0$ one gets an expression for the half-life τ, if η_0 refers to the upper limit of the momentum distribution curve. Dividing the first of these two equations by the second and multiplying by the total number of events being considered (=number of tracks measured, in this case) one gets for the number N of particles with momenta between $mc\eta$ and $mc(\eta+\Delta\eta)$

$$N\Delta\eta = Kf(Z, \eta)(C-(1+\eta^2)^{\frac{1}{2}})^2\Delta\eta, \qquad (1)$$

where K is a function of Z, τ and η_0; $C=(1+\eta_0^2)^{\frac{1}{2}}=E_0+1$. In this latter expression E_0 is the energy in mc^2 units of the upper limit; this we shall use instead of $(1+\eta_0^2)^{\frac{1}{2}}$ for typographical simplicity. Fermi has given the form of $f(Z, \eta)$ explicitly and it may easily be shown that it can be reduced to the expression

$$f(Z, \eta) = \eta^2\left(\frac{2\pi y}{1-e^{-2\pi y}}\right) \qquad (2)$$

for all light elements up to about copper ($Z=29$). In (2) $y = Z(1+\eta^2)^{\frac{1}{2}}/137\eta$. We may now write (1):

$$K(N/f)^{\frac{1}{2}} = C-(E+1) \quad \text{(Fermi)}. \qquad (3)$$

It is clear that if one plots $(N/f)^{\frac{1}{2}}$ against $(E+1)$ one should get a straight line if the Fermi theory is being obeyed. Furthermore the upper limit E_0 is given by the intercept C on the $(E+1)$ axis.

The modification of the Fermi theory introducted by Konopinski and Uhlenbeck merely changes the squared term in (1) into a fourth

[11] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **48**, 107 (1935).

power so that we may write for the equation of the distribution curve:

$$K(N/f)^{\frac{1}{2}}=C-(E+1) \qquad \text{(K–U)}. \qquad (4)$$

Treating the data in an exactly analogous way will yield through C the K–U upper limit.

Results

Carbon N^{13} (positron emitter)

We have studied the radioactivity induced in carbon by fast deuterons in both the forms of Acheson graphite and soot deposited on a thin piece of copper. The first source holder was used here. To get a sufficiently strong sample requires an exposure of only a few seconds to the deuteron beam (2 microamperes at 4 MV). To avoid contamination with O^{15} and F^{17} which would be formed in the surrounding air and driven by recoil onto the carbon target the target chamber was evacuated during bombardment and further the carbon was protected by a thin copper foil (0.0001″ thick). These precautions were found to be insufficient, for a freshly activated sample gave an upper limit which was around 200 kv higher than that got from a target which had been allowed to decay through eight half-lives of N^{13} (eighty minutes). Dr. S. N. Van Voorhis in some unpublished work has found a short period of between 2 and 3 minutes in carbon activated under these conditions. This is certainly the "air effect" due to occluded air in the carbon. Indeed the high energy tracks are all to be found in the early pictures taken of a freshly activated sample. The high energy tracks have been attributed to O^{15} (and perhaps to a much smaller degree to F^{17}) on the basis of some unpublished measurements on the positron spectrum of this element by Delsasso, Fowler and Lauritsen which they have kindly communicated to us. They find that the upper limit of O^{15} is slightly greater by an amount which is about the same as we find freshly activated carbon to exceed N^{13}.

The data on old samples of activated carbon are shown in Fig. 1 and refer to 723 tracks. Through the histogram has been drawn a smooth curve which is taken from the K–U theory by fitting a straight line to the linear plot as was discussed in the previous section. This linear plot is given in Fig. 2. On it one will notice that the intercept which we have denoted by C is not beyond the last measured track. This is not usual, as we shall see, but may easily be attrib-

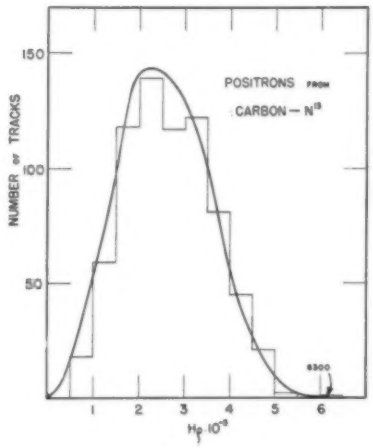

FIG. 1. Distribution histogram for the positrons emitted from activated carbon (N^{13}). Superposed on the experimental histogram is the K–U curve which has been fitted to it. The endpoint of this fitted curve is at $6300H\rho$ as indicated.

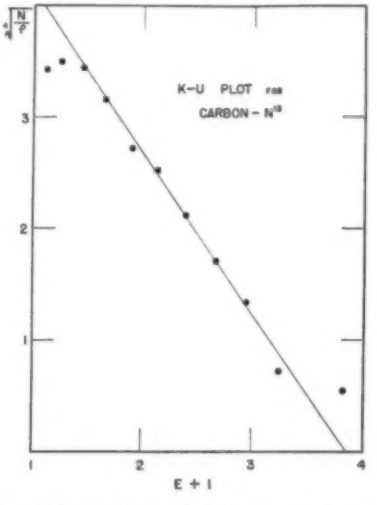

FIG. 2. K–U plot for activated carbon. Plotted in this way the points should fall on a straight line if the distribution curve has the shape demanded by theory. Deviations from linearity are here seen only at the low energy and where one always counts too few tracks, and at the high energy end where statistical fluctuations are important. The upper limit is here given by $E+1=3.8$ ($=6300H\rho$ $=1.45$ MV).

uted to errors in measurement. It is obvious from the form of (4), since the fourth root there occurring requires a high order of contact of the distribution curve with the axis of abscissae, that the theoretical curve may have an end point quite beyond the last measured tracks. It is not inconceivable that the true distribution curve should have this high order contact and that the measured "upper limit" should fall short of the true upper limit because of the inherent difficulties of following the curve to its limit. The hint that the curve might go on clinging to the axis for some distance beyond the last observed track has been given some foundation in the recent work of Scott[12] who has exhibited a curve for Ra E which passes beyond the accepted limit of $5500H\rho$. We feel some hesitancy in offering this as evidence since Scott's work was done with a magnetic spectrograph where scattering renders the counts in regions where particles are scarce very uncertain.

The evidence is good that the radioactive body in this case is formed by the following reaction

$$C^{12}+H^2=N^{13}+n^1+Q_1$$

and one must imagine it decaying according to

$$N^{13}=C^{13}+e^+ +(e^-)+Q_2.$$

A further reaction is known, namely,

$$C^{12}+H^2=C^{13}+H^1+Q_3.$$

From this set of reactions we may eliminate the heavy particles and write

$$Q_2=(H^1-n^1)+(Q_3-Q_1)-(e^+ +e^-).$$

The difference (H^1-n^1) is best taken from Feather's work[13] on the photoelectric disintegration of the deuteron and amounts to -0.48 MV. Bonner, Delsasso, Fowler and Lauritsen[14] have measured all the Q's under almost identical conditions and find $Q_1=-0.37$ MV, $Q_3=2.65$ MV and $Q_2=1.45$ MV. Taking these figures together with their probable errors we find that $Q_2=1.52\pm0.11$. From the intercept C of Fig. 2 we get $Q_2=1.45$ MV which is in excellent agreement with the result of Bonner, Delsasso, Fowler and Lauritsen and with the calculated endpoint. The agreement would not have been nearly as good if we had taken the upper limit at $5500H\rho(=1.25$ MV) which is at the point the histogram (Fig. 1) seems to indicate. It seems then very likely that the K–U theory offers a good description of the shape of β-ray distribution curves both as concerns the shape of the curve and as regards the endpoint.

Cockcroft, Gilbert and Walton[15] have published a distribution curve suggesting the existence of two groups of positrons, when carbon is activated with protons. It will be interesting to investigate this matter further inasmuch as the deuteron activation gives clearly a simple spectrum.

We have also examined the spectrum of carbon activated at energies below 1 MV. It might be expected that if the apparent simplicity of the N^{13} spectrum were due to the superposition of many elementary distributions due to different excited states of N^{13} one might alter this appreciably by the fourfold change in bombarding energy. No change was apparent in the distribution curve or the upper limit. This examination could only be made on a fresh sample because of the weak activity at the lower bombarding voltage, so the distribution curve was compared with that of another fresh sample activated at the high voltage. The upper limits obtained in these cases pertain largely to O^{15} and we can definitely say that any effect dependent on bombarding voltage is small for these elements.

Oxygen F^{17} (positron emitter)

H. W. Newson has recently shown[16] that a radioactive substance can be prepared by bombarding oxygen with deuterons and that this substance is probably F^{17}, the same element which has been prepared by Wertenstein from nitrogen with α-particles. Newson gives the half-life as 1.16 minutes. In our first source holder we have studied this activity which is also a positron emission, with a target prepared by placing a platinum foil in the deuteron beam

[12] F. A. Scott, Phys. Rev. **48**, 391 (1935).
[13] Report of the British Association discussion on nuclear physics at Norwich, Nature **136**, 467 (1935).
[14] T. W. Bonner, L. A. Delsasso, W. A. Fowler and C. C. Lauritsen, Berkeley Meeting, Am. Phys. Soc., Phys. Rev. **49**, 203 (1936).
[15] J. D. Cockcroft, C. W. Gilbert and E. T. S. Walton, Proc. Roy. Soc. **A148**, 225 (1935).
[16] H. W. Newson, Phys. Rev. **48**, 790 (1935).

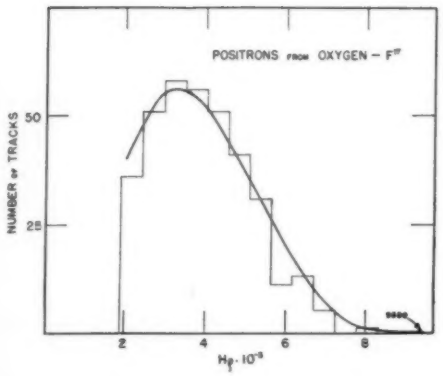

FIG. 3. Distribution histogram for the positrons emitted from activated oxygen (F^{17}) together with the theoretical K–U curve. The endpoint of the theoretical curve is at $9500H\rho$.

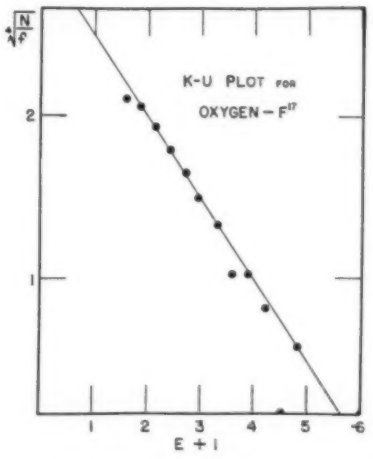

FIG. 4. K–U plot for activated oxygen. This plot is again linear and extrapolates to an upper limit at $E+1=5.7$.

when the target chamber was full of oxygen. A strong sample of F^{17} is driven by recoil onto the foil.

Fig. 3 shows the distribution histogram together with a K–U curve which has been fitted to it by means of the linear plot in Fig. 4. The upper limit indicated by the latter is 2.4 MV, while an inspection of the histogram would lead to about 2.1 MV. In this case we have again a triad of reactions such as was found in the case of carbon. These are:

$$O^{16}+H^2=F^{17}+n^1+Q_1,$$

$$F^{17}=O^{17}+e^+ +(e^-)+Q_2,$$

$$O^{16}+H^2=O^{17}+H^1+Q_3.$$

Newson estimates Q_1 as -1.8 MV. Cockcroft and Walton[17] have observed a group of protons with range 8 ± 1 cm when oxidized tungsten is bombarded with deuterons, which group vanishes when the tungsten is heated. If this group is to be attributed to oxygen according to the last of our three reactions we find $Q_3=1.8$ MV. As before $Q_3=-0.48+1.8+1.8-1.02=2.1\pm0.24$. This figure is in better accord with the value obtained from an inspection of the histogram than with that indicated by the K–U plot. The precision of the work on which Q_1 and Q_2 rest is not sufficiently great for this to be a matter of serious concern as yet.

[17] J. D. Cockcroft, Int. Conf. Physics, p. 124.

Sodium Na^{24} (electron emitter)

Radio-sodium has been extensively studied by E. O. Lawrence.[18] Substances of the same decay period have been prepared by Fermi and his collaborators[19] from magnesium and aluminum. M. C. Henderson[20] has reported the creation of the same substance from magnesium by deuteron bombardment.

We have examined the radiations emitted from this body by bombarding a target of sodium fluoride with deuterons. The activity due to the fluorine has a very short life of the order of ten seconds. The sodium targets are always "aged" before being put in the chamber. The data we have obtained from one target, bombarded for 1 hour at 1 microampere at 3 MV and aged for 4 hours were illustrated in an earlier communication from us on this subject[21] and reference is made to the figure there reproduced. That figure was based on 900 tracks, photographed with the source in our first holder. Since the time of that note we have carefully recalibrated our magnetic field as explained above and the figures for $H\rho$ of the upper limit there given must be lowered by four percent. This gives an upper limit (K–U) at $8000H\rho$ or 1.95 MV. By in-

[18] E. O. Lawrence, Phys. Rev. **47**, 17 (1935).
[19] E. Fermi, E. Amaldi, O. d'Agostino, F. Rasetti and E. Segrè, Proc. Roy. Soc. **A146**, 483 (1934).
[20] M. C. Henderson, Phys. Rev. **48**, 855 (1935).
[21] See footnote 7.

spection one would put this figure lower at, say, 1.7 MV.

In sodium we get tracks beyond the K–U upper limit but in view of the strong γ-radiation emitted from this body we attribute these tracks to the high energy component of this γ-radiation.[22]

There is some suggestion in the form of the histogram for sodium that there might be a low energy group of β-particles. Whether or not such exists is a subject for a future investigation.

Silicon S^{31} (electron emitter)

The production of S^{31} from silicon has been achieved by Fermi with neutrons and by Newson[23] with deuterons. The two authors give 145 min. and 170 min., respectively, for the half-life. The data given by us[22] earlier were obtained from a piece of quartz which had been bombarded with deuterons for 45 min. at 0.8 microamperes at about 3 MV. The sample was aged for 15 minutes. The momentum distribution curves are shown in the note referred to above. Again the magnetic field must be corrected to take account of our recalibration thus giving an upper limit at $8400H\rho$ or 2.05 MV, whereas the histogram apparently ends at about $7600H\rho$ (1.8 MV).

Phosphorus P^{32} (electron emitter)

Fermi first demonstrated that P^{32} could be made from sulphur and chlorine by neutron bombardment. Newson[23] then showed that the same substance could be made from phosphorus with deuterons, and recently Fahlenbrach[24] has produced it from the action of α-particles on silicon. The last two authors give 14.5 days and 14.0 days, respectively, for its half-life. Ambrosen[25] has made measurements on the energy distribution of the bodies produced from neutron bombardment and has shown them to be identical within the precision of his measurements. He gives 2 MV as the upper limit.

We have investigated the β-spectrum of P^{32} very carefully because it has seemed to be the ideal substance whereby to test the Konopinski-Uhlenbeck theory. It has a life which is so much longer than any other artificially produced radioactive body that contamination effects can be given ample opportunity to die out. Furthermore the only γ-radiation emitted is a soft Bremsstrahlung.

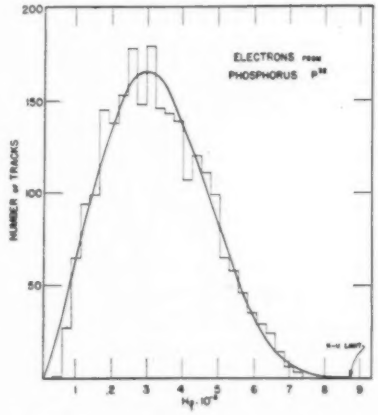

FIG. 5. Distribution histogram for the electrons emitted from activated phosphorus (P^{32}) with the fitted K–U curve showing an endpoint at $8700H\rho$.

Red phosphorus was activated in a current of 4.5 microamperes slightly under 4 MV for 2.5 hours. The sample was not used for these measurements until three days later, and then was found to be much too strong so a fragment of the 0.0001″ copper foil which had been used to protect the target chamber of the big accelerator from spattered phosphorus was found to carry sufficient activity for the measurements. This piece of foil was placed in the second holder—the experimental conditions being then as near ideal for an analysis of the shape of a distribution curve as possible. In the first place the target was sufficiently old to exclude contamination to a great extent; secondly, the β-rays were collimated so that the solid angle available was the same for all energies; thirdly, the target was thin and there was no backing except the wall of the chamber which was at least 2 cm distant so that the chance of reflection was very slight. Despite these good conditions the distribution curve differed very slightly from those we had been getting with the first source holder. On the basis of this we feel that at least for low energy spectra the first holder with the mica window on the brass grid is

[22] J. R. Richardson, Berkeley Meeting, Am. Phys. Soc., Phys. Rev. **49**, 203 (1936).
[23] H. W. Newson, Phys. Rev. **48**, 482 (1935).
[24] H. Fahlenbrach, Zeits. f. Physik **96**, 503 (1935).
[25] J. Ambrosen, Zeits. f. Physik **91**, 43 (1935).

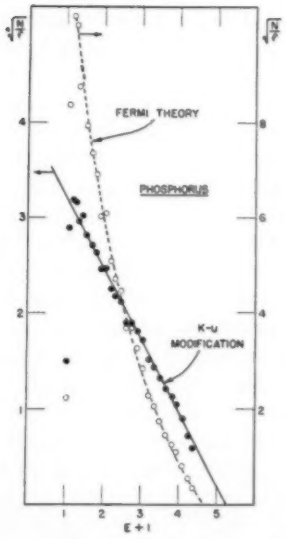

FIG. 6. The data given by the histogram in Fig. 5 are here plotted in two ways: The (black) points marked "K–U modification" should fall as they do on a straight line if the K–U theory is being followed. If the Fermi theory is being followed the (white) points should fall on a straight line as they clearly do not. This curved line has been found for all radioactive bodies when plotted in this way and indicates the character of the observed deviations from the simple Fermi theory. The K–U line shows an endpoint at $E+1=5.2$.

sufficiently satisfactory for upper limit determinations, and gives the shape of the distribution curves accurately above $2000H\rho$.

Fig. 5 shows a distribution curve containing 2372 tracks together with a theoretical K–U curve obtained from the linear plot in Fig. 6. The Fermi plot also shown in Fig. 6 is typical of the curves we began getting as soon as we substituted hydrogen for oxygen in the cloud chamber. Here the data have been plotted so that if Eq. (3) were being obeyed the line marked "Fermi theory" should be straight. The concavity shown here is to be found in all of our hydrogen-chamber distributions and is characteristic of the observed deviations from the Fermi theory. On the other hand the line marked "K–U modification" should be straight if the K–U theory is being followed. The plotted points have had a straight line drawn through them and we regard the closeness with which the points cluster around this line as a measure of the very excellent way in which this theory accounts for the shape of a β-ray distribution curve. The fact that the K–U line in Fig. 6 cuts the axis beyond the last experimental point has been a strong argument against the acceptance of the upper limits given by fitting the data in this way. However, it must be realized that the intensity in this extended tail is very small. Thus in the present case the chance of finding a track with a momentum of $8400H\rho$ can be shown on the basis of the K–U curve which we have fitted to our data to be about 1 in 4000. Also the complete agreement between the upper limit of the positron spectrum from N^{13} and the energy computed from other reaction energies makes it seem very certain that these K–U upper limits are the significant endpoints from the standpoint of energy balances.

The upper limit indicated by the K–U plot is at $8700H\rho$ (=2.15 MV) whereas one would estimate from the histogram at about $7500H\rho$ (=1.8 MV). Either figure is in good agreement with the limit of 2 MV found by Ambrosen.

Paxton[26] has reported the emission of positrons from a target of deuteron activated red phosphorus. The half-life of the active body in this case is around 50 hours and so is not associated with the electronic emission from P^{32}. A more detailed account of this work will be published shortly.

Chlorine (electron emitter)

An active chlorine was prepared by Fermi by bombarding chlorine with neutrons. What is presumably the same body has been made in this laboratory and various workers have given the half-life values ranging from 39 min. to 40.5 min. as compared with that found by Fermi of 37 min. A sample of this substance was prepared by bombarding AgCl, fused on a piece of silver, with about one microampere of 3 MV deuterons for 20 minutes. This was examined in the first source holder and a curve obtained which was very strongly asymmetric showing a strong maximum at around $2500H\rho$ and an endpoint beyond $18{,}000H\rho$. We had always got such curves from chlorine before and thought that the grid on which the mica window was supported might be slowing fast electrons thus causing them to be grouped among the slower ones. A K–U plot of

[26] H. C. Paxton, Berkeley Meeting, Am. Phys. Soc., Phys. Rev. **49**, 206 (1936).

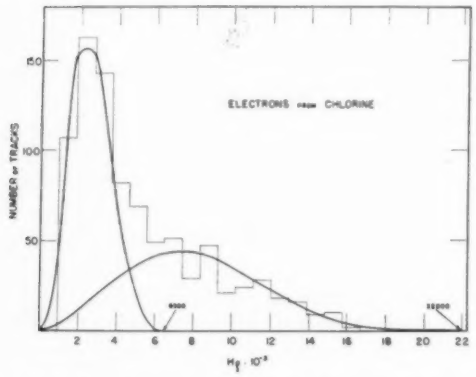

FIG. 7. Distribution histogram for the electrons emitted from activated chlorine. The general shape of this curve is much more skew than a simple K–U curve. As can be seen from Fig. 8 one may resolve this curve into the sum of two simple K–U curves. The upper limits so found are at $6500 H\rho$ and $22{,}000 H\rho$.

the data was found to be resolvable into two, suggesting that radio-chlorine might either consist of two separate active bodies with the same half-life or that its spectrum might be the sum of two simple P^{32}-like spectra, as Ellis and Mott have suggested for the heavy radioactive bodies.

This point is so interesting that we activated another sample and allowed it to age for three hours before mounting it directly in the chamber with no grid in front of it. The distribution curve for this case is shown in Fig. 7 and the K–U plot in Fig. 8. Here again the possibility of separating the curve of Fig. 7 into the sum of two was again apparent. When this was done upper limits of the two groups were indicated at the same points as had been obtained in the preceding case when the brass grid was in place. The relative proportions of the high and low energy groups with the grid was roughly 2 : 1 and without the grid 1 : 1. We feel that this is probably to be accounted for by the fact that the absorption curve is much steeper for electrons of the energies which make up the lower group than for the upper group.

Feeling that the legitimacy of such a resolution of the data into groups was perhaps questionable we did not pursue the matter further. It was after this that the data on phosphorus and the agreement of the reaction energies in the case of N^{13} added so much weight on the side of the K–U theory. Such a separation now seems to be a perfectly valid consequence of the apparently good accord between theory and experiment, and we plan to investigate this matter more carefully later.

If this resolution is right we may think of active chlorine decaying in one of two ways to form a stable argon. There should then be γ-rays of energy equal to the difference. The K–U plot in Fig. 8 gives the two β-ray limits as 1.5 and 6.1 MV. The difference is 4.6 MV. Richardson[22] has reported preliminary data which indicate two γ-lines at 2.4 and 1.9 MV. It might be that the transition from the excited level at 4.6 MV in argon to the ground state might be accomplished in two groups of $2.4+1.9=4.3$ MV. Unfortunately Richardson finds these lines to differ in intensity, but the data are as yet not sufficiently good to be sure the above explanation is invalid. Another alternative is to assign one group to Cl^{36} and the other to Cl^{38} attributing the fact that the periods are identical to coincidence. Again one might imagine the two groups being due to two successive disintegrations of which the second had a period short compared to the first. Against this may be raised the objection that if the active body is really a chlorine isotope then any argon isotope which could result is stable. None of these are very satisfactory and so we defer the question until more data has been accumulated.

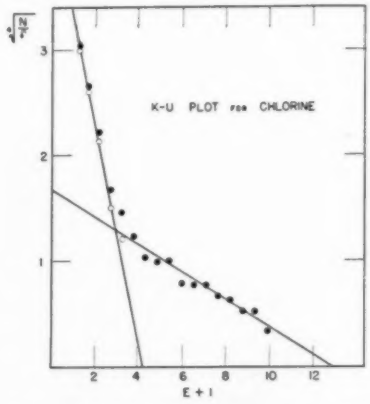

FIG. 8. K–U plot for activated chlorine. The black points are the experimental data. The presence of two linear portions to this curve is apparent. The lower group is indicated by the white points which are got by subtracting the "background" due to the lower energy part of the upper group. Endpoints are given by $E+1=4$ and 13. (The scale of abscissae in this figure has not been changed to conform to our recalibration of the magnetic field and so shows endpoints higher than the ones quoted.)

It is possible that such a presumably composite spectrum as this would exhibit a dependence on bombarding voltage. The opportunity to examine this has not arisen as yet.

Argon A^{41} (electron emitter)

That activity could be excited in argon by deuterons was discovered by A. H. Snell[27] who gives it a period of 108 minutes. Dr. Snell has prepared for us sources of this body by recoil on a platinum foil immersed in an atmosphere of argon through which the deuteron beam was passing. Photographs were taken with the foil mounted in the second source holder. The results of these photographs are shown in Fig. 9 as a distribution histogram. This has a shape very similar to the curves found for chlorine and indeed the K–U plot in Fig. 10 suggests that here again one might think of this curve as being decomposable into two. In this case it seems that the data are too few to warrant setting a value for the upper limit of the faster group, except to say that it is in the neighborhood of 18,000$H\rho$ (=5 MV) according to the K–U plot. The lower group would fall at 6600 $H\rho$ (=1.5 MV). These seem to be unrelated to the γ-ray at 1.3 MV which Richardson[22] has found, but it is a little premature to discuss this too fully until more complete data are available.

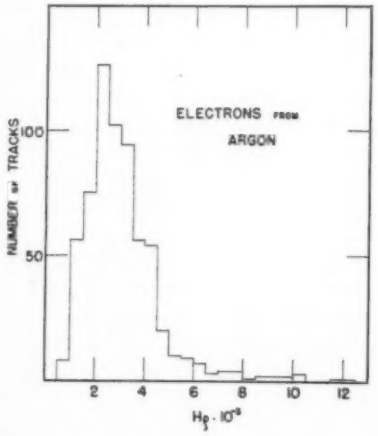

FIG. 9. Distribution histogram for the electron emitted from activated argon (A^{41}). This curve also shows evidence in its skewness of the existence of a high energy group. The data on this upper group are not abundant enough to warrant the drawing of theoretical curves on this histogram.

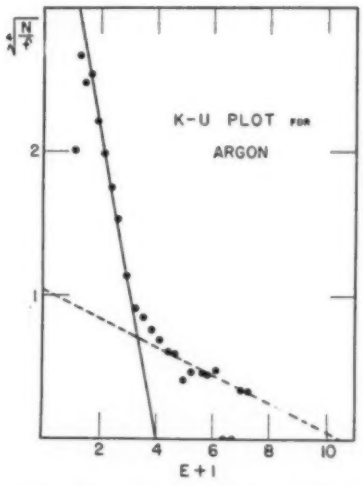

FIG. 10. K–U plot for activated argon. The lower group has an upper limit at $E+1=4$ (=1.5 MV) and the tentative trend of the upper group has been indicated by the dotted line. The endpoint of this upper group is around $E+1=10.5$ (=5 MV).

Potassium K^{42} (electron emitter)

This element was first prepared from potassium by Fermi with neutrons and later by Hevesy[28] from scandium with neutrons and in this laboratory with deuterons. This is a substance for which we also found evidence of two groups in the oxygen filled chamber. The target responsible for the 870 tracks whose momentum distribution is shown in Fig. 11 was potassium fluoride fused on a piece of platinum gauze. It was activated for one hour with one microampere of about 3 MV deuterons. In Fig. 11 we have again resolved the distribution into two groups as before. The hint to do this is contained in Fig. 12 where the K–U plot is fitted very closely by two straight lines and not at all by one. The upper limits are at 6300$H\rho$ and 16,100$H\rho$. The lower group is estimated from these data (which were taken with the first source holder, and hence are not greatly to be trusted) at half the intensity of the upper group. We have attempted to investigate the rather weak γ-radiation emitted by this element but have not

[27] A. H. Snell, Berkeley Meeting, Am. Phys. Soc., Phys. Rev. **49**, 207 (1936).

[28] G. Hevesy, Nature **135**, 96 (1935).

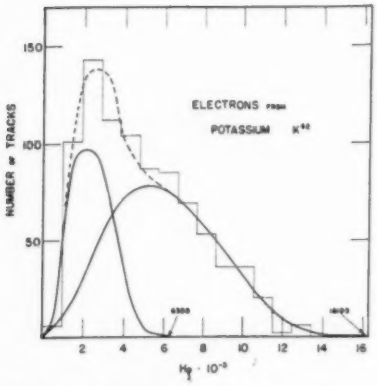

FIG. 11. Distribution histogram for the electrons from activated potassium (K^{42}). Here too the curve is too skew to be fitted by a single K–U curve but can be resolved into two as shown. The dotted line represents the sum of these two curves, and conforms very well to the histogram.

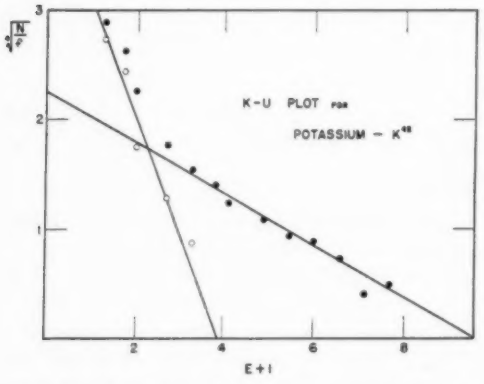

FIG. 12. K–U plot for activated potassium. This has been resolved into two lines by subtracting the "background" of the upper group to give the white points which then represent the lower group. The lines indicate upper limits for the two groups at $E+1=3.8$ and 9.6.

as yet succeeded in preparing sources of sufficient strength.

DISCUSSION

The data given above indicate that the Konopinski-Uhlenbeck theory gives a very good account of the shape of the distribution curves of the β-rays from the light radioactive elements. We have cited cases of three electron emitters (Na^{24}, Si^{31}, P^{32}) and two positron emitters (N^{13}, F^{17}) where deviations from the theoretical shape of the curve of the observed points are surprisingly small. The spectra of the three elements Cl, A^{41}, K^{42} can be resolved into two components each of which is very closely a K–U curve. This type of resolution is very similar to that proposed by Ellis and Mott[29] and does not seem to be completely unreasonable. The validity of such a resolution can perhaps only be tested when the energies and intensities of the γ-radiations from such bodies have been studied more closely, and also when more precise determinations of the β-ray endpoints have been made.

In fitting Eq. (4) to the data in any particular instance it is to be noted that although there are two constants appearing, only one is really adjustable, the other being determined by the number of tracks being considered. Also it is impossible to adjust the constant C in any way other than that we have used—any other adjustment, such as identifying it with the highest energy actually measured, will lead to a poorer fit between experiment and theory.

The high upper limits predicted by the K–U theory present a new difficulty in the study of the β-rays. If the order of contact with the axis is really as high as indicated by theory we must consider all the data hitherto gathered about upper limits as inexact. This would be very unfortunate, but the evidence seems strongly to indicate that it may be true. The very excellent agreement of the reaction energies in carbon found by Bonner, Delsasso, Fowler and Lauritsen and also by ourselves constitute the first direct information on this point. It may be that one must discount the fine correlation of the energies of the two branches of the Th C bodies obtained by Henderson[30] since it rests on two β-ray measurements whose upper limits are not greatly different. It is difficult to judge in this case since the authors give no experimental points on their curves by which one might estimate the chance that they could miss a tail with a high order of tangency to the momentum axis. Further evidence that such tails may really exist is to be found in the work of Scott mentioned above. Scott has carefully investigated the spectrum of Ra E and gives a higher upper limit than has previously been obtained. His data when plotted

[29] C. D. Ellis and N. F. Mott, Proc. Roy. Soc. **A141**, 502 (1933).
[30] W. J. Henderson, Proc. Roy. Soc. **A147**, 572 (1934).

in the form of Eq. (4) show the linear form very well and from the value of C indicated by them one deduces an upper limit very close to the one Scott gives.

Attempts have been made to fit the data of this paper on Sargent[31] curves. For the light radioactive bodies it would seem that one can at best say that no striking regularity is thus exposed.

The data presented above are summarized in Table I.

The authors are very much indebted to Professor E. O. Lawrence and Professor J. R. Oppenheimer for many discussions of this work.

[31] B. W. Sargent, Proc. Roy. Soc. **A139**, 659 (1933).

TABLE I. *Summary of values for the upper limit of β-ray spectra from various elements.*

SUBSTANCE	TARGET	UPPER LIMITS INSPECTION	UPPER LIMITS K–U LINES
$N^{13}+$	Acheson graphite soot	1.3 MV	1.5 MV
$F^{17}+$	O_2	2.1	2.4
Na^{24}	NaF	1.7	1.95
Si^{31}	quartz	1.8	2.05
P^{32}	red P	1.8	2.15
Cl	AgCl	4.8	1.5 6.1
A	A gas	2.7	1.5∽5
K	KF	3.5	1.4, 4.4

We are also very grateful to the Research Corporation and the Chemical Foundation and the Josiah Macy, Jr. Foundation for grants whereby these experiments were made possible.

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The Ionization Gauge for Atomic Beam Measurements

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Use of two types of current measuring devices, galvanometer and vacuum tube amplifier, with an ionization gauge shows that the stable sensitivity,

$$\left(\frac{\text{positive ion current/mm gas pressure}}{\text{current fluctuation}}\right),$$

is greater for low electron emission densities thus favoring the use of the amplifier with its greater current sensitivity. Such factors as electrode size, relative potentials, electron emission, interelectrode leakage, type of inlet, gas adsorption, surface charges, temperature changes, B.K. oscillations, and variations of pumping speed which influence the design of a gauge are discussed and an arrangement to fulfill the necessary requirements is described. Performance tests show the gauge to be capable of measuring beam pressure changes at least as small as 3×10^{-11} mm of mercury or approximately 3×10^{9} mercury atoms per square centimeter per second.

THE properties of the ionization gauge method of measuring atomic beams are such that it would enjoy a wider field of application if the sensitivity were sufficient. Use by Zahl and Ellett[1] has shown no indication of the limitation attributed to adsorption phenomena by Johnson[2] but rather has indicated that the gauge might be capable of measuring beam pressure[3] changes of less than 10^{-10} mm of mercury. This, if true, would extend the range of usefulness to such a degree that a more careful investigation seemed advisable.

ANALYSIS OF MEANS OF EXTENDING SENSITIVITY

In normal use the sensitivity of an ionization gauge is determined by two factors, the magnitude of the electron current and the sensitivity of the positive ion current measuring instrument. Thus, to improve sensitivity, one may increase either the electron current or the sensi-

[1] Zahl and Ellett, Phys. Rev. **38**, 977 (1931).
[2] T. H. Johnson, Phys. Rev. **31**, 103 (1928).
[3] Unless otherwise indicated, all references to sensitivity are based on the effective pressure of the beam, expressed in millimeters of mercury, at the gauge inlet. Effective beam pressure is defined as that pressure inside a closed vessel which makes the number of molecules striking unit area of bounding surface in one second equal to the number transported by the beam across unit normal area in an equal time.

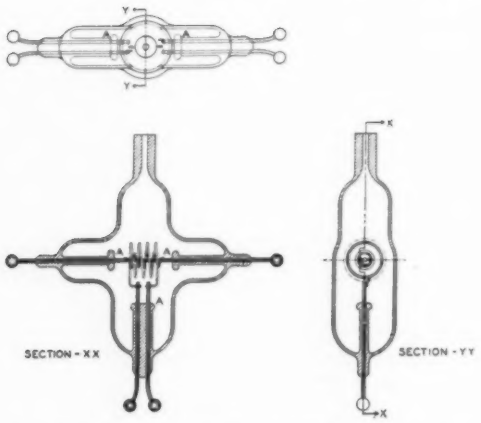

FIG. 1. Details of ionization gauge for beam measurements. Plate coil, 4 turns 8 mm dia. 0.015″ wire, length 7 mm; grid coil, 5 turns 3 mm dia. 0.010″ wire, length 7 mm; filament coil, 25 turns ½ mm dia. 0.005″ wire, length 7 mm.

tivity of the positive ion current meter until the critical ratio

$$\left(\frac{\text{positive ion current/mm gas pressure}}{\text{positive ion current fluctuation}}\right)$$

becomes too low. A fluctuation not exceeding ½ mm on the recording galvanometer, for an arbitrarily selected range, is considered satisfactory.

When measuring positive ion currents with a galvanometer having a constant of 4×10^{-11} amp./mm, the electron current can be increased to about fifteen milliamperes before fluctuations arising from sources other than the shot effect exceed the limit. The corresponding beam pressure sensitivity is of the order of 4×10^{-10} mm of mercury. The limiting factor seems to be instability in the thermionic emission.

The alternative, increased sensitivity of the positive ion current detector, remains. But this evidently is of no advantage unless the relative variations inherent in the first method can be reduced. Fortunately a decrease in filament temperature reduces the fluctuations in electron current more than the current itself; that is, the ratio $\Delta i/i$ becomes smaller. Hence, because of the linear relation between positive ion current and electron current, a decrease in electron current should permit a more than equivalent increase in the sensitivity of the positive ion meter with a corresponding net gain in performance. The beam sensitivity can be increased in this manner to a limit of approximately 2×10^{-11} mm of mercury at which point other difficulties become more important; but a recording device with a constant of the order of 10^{-13} amp./mm is necessary.

The design of a gauge to detect such small changes of pressure and to operate in conjunction with a high sensitivity current indicator requires consideration of the influences of various factors which will be taken up under the following heads: (1) shape and size of electrodes, (2) surface leakage (electrical), (3) potential of the electrodes, (4) instability in thermionic emission, (5) inlet resistance, (6) residual pressure in gauge, (7) Barkhausen-Kurtz oscillations, (8) effective ionizing path, (9) surface charges, (10) zero drift. In the following section each of the above factors will be discussed and the provision made for it in the gauge described. It is evident that the subjects are not mutually exclusive, but the division is a convenient one from an experimental standpoint. All references are to the gauge shown in Fig. 1.

GAUGE DESIGN

Electrodes

Fortunately, extensive investigation on the shape, size and position of the electrodes has been completed by Dushman and Found.[4] They find certain general forms to be better than others but the superiority lies more in stability and in linear response than in sensitivity. The general arrangement of electrodes they suggest has been maintained but certain revisions have been necessary for adaptation to beam measurements.

Gauges differing by a factor of two in electrode size and spacing showed little difference in sensitivity for low pressures and equal electron currents.

It will be noted that the collector electrode, as well as the other electrodes, is a tungsten spiral and that all electrodes have two leads sealed in. This permits outgassing of the individual elements by passing current directly

[4] Dushman and Found, Phys. Rev. **17**, 7 (1921).

through them. In this manner bursts of ionization resulting from the sudden release of gas in the usual bombardment heating are avoided. Use of a spiral collector electrode eliminates much of the evaporated oxide and metal arising from the outgassing of a cylinder and by reducing the surface decreases the amount of gas evolution during operation.

Sagging of the middle turns of the collector electrode during outgassing causes some difficulty if the coil is too closely wound. The inner turns get hot enough to sag before the outer ones are heated sufficiently. It was found that a spacing of 2 mm per turn with 0.015-in. wire was sufficient for coils up to 1.5 cm in diameter. End loss keeps the leads sufficiently cool to prevent any sagging of the element as a whole.

Surface leakage

It is necessary that the leakage across the gauge walls from all elements to the collector be kept small. Such leaks are usually very unsteady and would serve to mask entirely the small positive ion current changes. This means that the collector (plate) leakage resistance must be of the order of 10^{13} ohms. A sufficiently high resistance was introduced by sealing the collector leads through the envelope as far as possible from all other elements and by putting glass beads at points A (Fig. 1) to shield parts of the envelope from material driven off during the outgassing of the elements. The resistance in actual operation was 10^{15} ohms.

Potentials of electrodes

Although the potentials applied to the collector and grid are not critical, there are optimum values. The variation of positive ion current with collector potential is of the general form shown in Fig. 2, which would lead one to attempt operation at the maximum point. However, the critical ratio becomes too low before the maximum is reached and the instability is exceedingly great after the peak is passed (collector potential of about two volts).

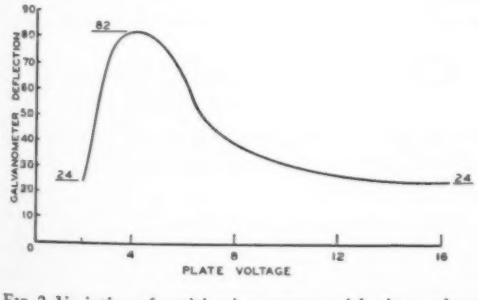

FIG. 2. Variation of positive ion current with plate voltage.

The increase in ion current can be attributed to change in space charge conditions and to lengthening of the effective ionizing path of the electrons. The smaller collector potential permits partial saturation of the positive ion current and also reduces the reverse potential gradient outside the grid. Both of these combine to allow the electrons to travel farther past the grid and produce more ions.

The increased instability could arise from electrons reaching the collector, the initiation of Barkhausen-Kurtz oscillations or the production of charges on the envelope.

If the above reasoning is correct, similar effects ought to be produced by an increase in grid potential, since it, too, would increase the length of path. It is found experimentally that grid potentials of more than 70 volts (collector, minus 20) introduce instability.

The conclusion is that one should use as high grid potential, or as small negative collector potential, as is consistent with stability and insulation.

Instability of thermionic emission

The reduction of instability ($\Delta i/i$) with temperature has already been mentioned. There remains the simple problem of choosing a filament with enough surface to give adequate emission at a low temperature with small power input. The resistance should also be high enough to make small contact variations negligible.

The filament shown, consisting of 25 turns of 5-mil tungsten wire formed into a 7-mm spiral by winding on a 15-mil wire as a form is sufficiently stable when operated by a storage battery with all connections carefully soldered. The emission should not exceed two milliamperes.

Inlet resistance

The gauge as normally used has a canal inlet which offers to the beam an impedance equal to that of a hole in a thin plate but resists the

escape of gas from the gauge with the full resistance of a tube. Experimental evidence[5] shows that the increase in sensitivity gained in this manner may be computed from Knudsen's laws of flow for gases at low pressure.[6] It must be remembered, however, that too great an increase in the resistance of the inlet will increase the recovery time of the gauge. If R is the Knudsen resistance of the canal and V is the gauge volume, the time constant of the gauge is RV. In practice V was from 6–10 cm^3 and R could be large enough to increase the sensitivity by a factor of ten to fifteen without causing RV to become unduly large (3–5 seconds).

Residual pressure in gauge

It is evident that the resistance of the inlet will maintain the residual pressure in the gauge at a value above that of the detector chamber and also increase the time to outgas the gauge for the preliminary clean-up. The effect of this resistance was determined by so arranging the set up that a ground glass plug with a bore 1.5 mm in diameter and 8 cm long could be removed at will. A beam was not used. As would be expected, the insertion of the plug caused an increase in pressure. The magnitude of the increase depended upon the previous treatment of the filament and outgassing of the gauge. The change accompanying insertion or removal of the plug decreased as the gauge was kept in operation and finally amounted to an alteration by a factor of about two.

Subsequent experience indicated that the use of a removable plug shortens the time of initial outgassing but does not reduce the final pressure enough to justify its use.

Barkhausen-Kurtz oscillations

Since the gauge is operated under conditions similar to those for producing B.K. oscillations, it is possible that the gauge might serve as a high frequency generator while in use. If so, it would be unstable because of its sensitivity to all changes of capacity in the vicinity.

The reactance introduced by the solenoidal electrodes should hinder the generation of these ultra-high frequencies. Also the observed stability accompanying operation at recommended potentials makes it seem unlikely that oscillations were present.

Effective ionizing path

Obviously any means which will serve to increase the electron path within the gauge should increase the number of positive ions per milliampere of electron current. It is not feasible, however, to achieve this with increased volume.

An attempt to increase the sensitivity by placing the gauge in a magnetic field showed but little effect, about 10 percent, for a field strength of 150 gauss. Increasing the field to 270 gauss decreased the effect to zero. The decrease in electron current because of increased space charge offset the gain in number of positive ions per milliampere.

One can also increase the path by using a smaller negative potential on the collector which allows the electrons to travel farther past the grid before being stopped. This has been discussed previously (electrodes).

Surface charges

It was thought that the open nature of the electrodes might introduce instability from surface charges, but none appeared which could be attributed to this source, while operating with recommended potentials.

Zero drift

A serious disadvantage of the ionization gauge is its lack of compensation for zero drift. The shift in zero point may arise from any of several sources, namely, changes in filament heating current, changes in work function of the filament, changes in thermal emissivity of the filament, changes in temperature of the bulb and surroundings, and changes in the residual pressure of the system.

Filament current changes can be reduced to such a point that they will have less effect than disturbances arising from other sources. A good storage battery and carefully soldered connections are essential.

Changes in work function and thermal emissivity can be corrected by maintaining the electron current at a constant value. This can be accomplished by altering the filament current

[5] Ellett and Zahl, Phys. Rev. **37**, 1112 (1931).
[6] M. Knudsen, Ann. d. Physik **28**, 75, 999 (1909).

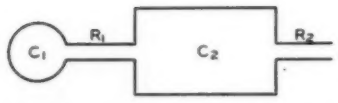

FIG. 3. Schematic diagram of detector chamber and pumping system.

either automatically or by manual adjustment, which is sufficient for most purposes.

Any shift in the temperature of the gauge will have a dual effect. It will change the equilibrium pressure built up by the canal inlet and will cause a change in electron emission. For convenience the gauge was mounted outside the system with only the canal inlet opening into the detector chamber. This arrangement should make temperature fluctuations larger than they would be with the gauge inside the vacuum system. Yet shielding from drafts removed all instability that could be attributed to this source.

Changes in residual pressure within the system are the most troublesome. Shifting of the level of liquid air in the traps is followed consistently by the gauge, as are fluctuations in pumping speed.

Theoretical consideration shows that reducing the fluctuations by means of a reservoir and connecting tubes of high resistance can only be accomplished with too great loss of pumping speed. Suppose, for example, a system of the type shown in Fig. 3 is used. C_1 is the detector chamber and R_2 is the combined resistance of the pump and connecting tube. We thus suppose R_2 to exhaust into zero pressure.

The differential equation for P_1, the pressure in C_1, is

$$C_2R_1C_1\frac{d^2P_1}{dt^2}+\left[C_2+C_1+\frac{R_1}{R_2}C_1\right]\frac{dP_1}{dt}+\frac{P_1}{R_2}-\left(1+\frac{R_1}{R_2}\right)Q_1-Q_2=0. \quad (1)$$

Q_1 and Q_2 are, respectively, the gas evolved from the walls of the chambers C_1 and C_2. Q_1 contains also the gas admitted by the beam. Solving (1) for the steady state gives the equilibrium value of P_1 which is

$$P_1=[R_1+R_2]Q_1+R_2Q_2. \quad (2)$$

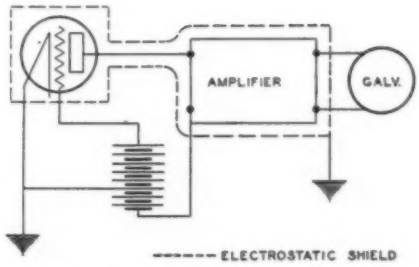

FIG. 4. Amplifier-gauge circuit.

Suppose R_2 abruptly changes to a value R_3, as would be the case if the pump speed changed. Using the value of P_1 from (2) as the initial pressure for the solution of (1) with R_2 replaced by R_3 and assuming $dP_1/dt\,|_{t=0}=0$ we find

$$P_1=-\frac{[R_3-R_2][Q_1+Q_2]}{m_1-m_2}[m_1e^{-m_2t}-m_2e^{-m_1t}]+P_{11},$$

where $m_1=[-A-(A^2-4B)^{\frac{1}{2}}]/2,$

$m_2=[-A+(A^2-4B)^{\frac{1}{2}}]/2$

and $A=(R_2C_1+R_2C_2+R_1C_1)/R_1R_2C_1C_2,$

$B=1/R_1R_2C_1C_2,$

$P_{11}=R_3[Q_1+Q_2]+R_1Q_1.$

Choosing the following values of the constants

$V_1=500$ cm^3 $\quad V_2=2000$ cm^3
$R_1=R_2=10^{-3}$ (a 1×10 cm tube approximately)
$R_3-R_2=10^{-5}$.

We find P_1 to have approximately the value

$$P_1=-10^{-5}[Q_1+Q_2][1.17e^{-.382t}-0.17e^{-2.62t}]+P_{11}.$$

Thus the time constant of the circuit is less than three seconds. The values of R_1 and R_2 are such that a pump speed of 20 liters per second will be reduced to about $\frac{1}{2}$ liter per second. The attainment of a 10-second response time would evidently seriously impair the pumping speed.

PERFORMANCE

The high sensitivity current measuring device used in connection with the gauge was a direct-current amplifier with a sensitivity of 1.56×10^{-14} amp./mm. It employed a type FP-54 vacuum

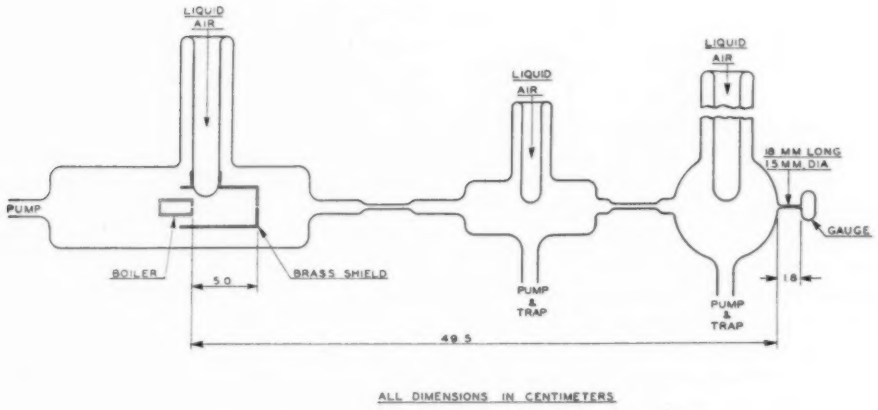

FIG. 5. Beam system for sensitivity measurements.

tube in a self-balancing circuit[7] and was connected to the gauge as shown in Fig. 4.

A beam system, as sketched in Fig. 5, served to make the necessary sensitivity measurements. Calculations on the basis of the boiler aperture, 0.016 inch diameter by 0.110 inch long, and the beam path, 49.5 cm, gives 1.68×10^{-7} as the ratio of beam pressure at gauge inlet to boiler pressure. The boiler aperture was treated as a hole in a thin plate, an approximation valid for all points on the axis of the aperture at distances large compared to its dimensions. The beam was interrupted when necessary by a magnetic shutter placed between the boiler and its liquid-air cooled shield.

With a pressure of 0.0023 mm of mercury in the boiler corresponding to an observed boiler temperature of 28°C the gauge gave consistent readings of 18 mm with an instability of less than 1 mm and a relaxation time of about two minutes. This corresponds to a beam pressure sensitivity of 2.15×10^{-11} mm of mercury or its equivalent, 3.1×10^{9} atoms per second per square centimeter.

The experimental points in Fig. 6 show a typical response to a mercury beam. The dotted curve represents the theoretical response of a gauge with a time constant of 3 seconds, the computed value for the inlet used. It is evident that adsorption phenomena are playing a part in the response but that they do not influence the accuracy.

These results were obtained with the amplifier at one-tenth maximum sensitivity. An attempt was made to use the maximum amplification without success. Variations in pumping speed and the shifting of the level of liquid air in the traps introduced troublesome pressure fluctuations.

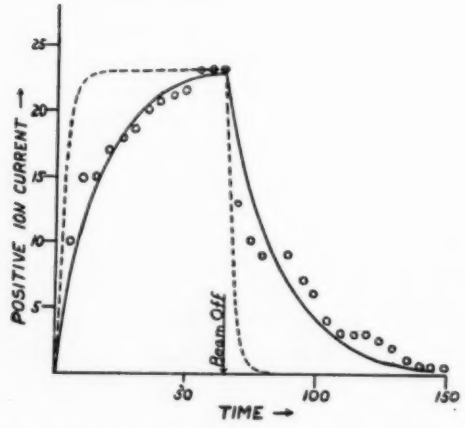

FIG. 6. Response to a beam of mercury. Circles, experimental points; solid curve, $y=23[1-e^{-t/15}]$ $0<t<65$ $y=23e^{-t/15}$ $65<t<150$; broken curve, $y=23[1-e^{-t/3}]$ $0<t<65$ $y=23\,e^{-t/3}$ $65<t<150$.

[7] Hart Brown and L. DuBridge, Rev. Sci. Inst. **4**, 533 (1933).

Operating Notes

Baking procedure

Before being used, the gauge was baked at 350°C for 24 hours; the traps on the pumps were filled with carbon dioxide and baking continued for 3 hours at 380°C. If at any time the carbon dioxide traps were allowed to warm up, the process was repeated.

Preliminary cleaning of gauge

Safe flashing currents for the elements in the gauge shown are: 7 amperes in plate coil, 4 amperes in grid coil, and 2 amperes in filament.

The first flashing of the gauge elements after construction usually deposits a heavy film of tungsten oxide on the walls and causes a leak from grid to filament. This can be removed by treatment with aqua regia. Unless the source of potential supplying the grid is poorly regulated, the grid-filament leak should cause little difficulty and perhaps need not be removed.

Good vacuum conditions are important; all oxygen and water vapor must be removed before the initial flashing is started and none admitted during operation. Otherwise the filament will evaporate in addition to becoming poisoned.

Equilibrium

Several hours are required for equilibrium to be established after the traps are filled and the gauge lighted.

Optimum potentials

The equilibrium is sensitive to voltage changes and it is necessary to provide a constant source for each. The values found most suitable were +67 volts on the grid, −22 on the collector and sufficient filament current to give 2 milliamperes electron emission.

Magnitude of positive ion current

For mercury, positive ion current in amperes is of the order of $Pi/100$ where P is the pressure in millimeters and i is the electron current in milliamperes. This means that the change in plate current is about 1μ amp. for a change in beam intensity of 10^{10} atoms of mercury per second per square centimeter when using an inlet 1.5 mm in diameter and 2 cm long.

Note on the Use of Two Gauges

One naturally thinks of balancing two gauges in a bridge arrangement to neutralize zero drift and thus permit greater sensitivity. When using a galvanometer in the collector circuits such an arrangement is feasible. However, a little consideration shows it to be less sensitive than the single gauge and no more stable provided the single gauge is operated at an equal sensitivity and care is used in the construction of the vacuum system.

Balancing two gauges with amplification introduces difficulty. Two choices are available.

If one amplifier is to be used, either the whole unit with its supply batteries must be insulated from ground with a resistance greater than 10^{11} ohms or else the gauge and its batteries must be so insulated. In either event the whole arrangement would have to be shielded completely and thus become unwieldy.

If, on the other hand, one seeks to use two tubes in the amplifier, one is faced with the problem of balancing the circuit and at the same time compensating for the lack of symmetry in the two gauges by adjusting the sensitivity of each tube. Although this can be done, it seems too troublesome to lead to advantageous use of the gauge.

Conclusion

The ionization gauge can be used to measure beam pressure changes of the order of 10^{-11} mm of mercury or its equivalent, a change in beam intensity of 3×10^{9} atoms per square centimeter per second.

The use of two gauges to balance out unavoidable zero drift and thus permit greater sensitivity seems possible only at the expense of complicated or unwieldy apparatus.

There is little point in attempting to increase the sensitivity without improving the vacuum conditions or making provision for neutralizing residual pressure changes.

Absolute Values of the Electron Mobility in Hydrogen

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(Received November 22, 1935)

A new method is described for the measurement of electron mobilities in gases. An electrical shutter method is employed in which the shutters take the form of two fine wire grids, alternate wires of which are connected to a high frequency alternating potential. Electrons pass through the grids only when the potential between adjacent grid wires is zero, and only electrons which cross the gas space in one half-cycle are received at the collecting electrode. A sharp maximum is thus observed in the electrometer current when the drift velocity of the electron multiplied by the time of one half-cycle is equal to the distance between the grids. The theoretical shape of the current curve is compared with experiment and good agreement observed. Measurements have been made between X/p of 0.03 and 20, and the values obtained are compared with the Compton mobility equation.

EARLY in the study of phenomena associated with the conduction of electricity in gases, it was observed that under certain conditions and in certain gases, the drift velocity of the negative carriers reached extremely high values. These velocities were of the order of a thousand times those which could be ascribed to carriers of molecular dimensions, and were correctly explained by Franck and Townsend as a characteristic of free electrons moving through a gas. Since these observations various measurements have been made of the drift velocity or mobility of electrons. In general, however, the results of different observers, while of the same order of magnitude, have not been in particularly good agreement. Furthermore the results have not been sufficiently accurate to permit any theoretical test to be made with confidence.

The importance of determining absolute values of electron mobilities in gases is twofold. Their magnitude is intimately associated with many phenomena involved in the electrical breakdown of gases, and a knowledge of its value is essential for the application of theory to experiment. In addition the drift velocity of an electron through a gas under the influence of an electrical field furnishes an excellent test of theories of electron scattering, collision, and energy transfer. Since most direct experimental observations of these quantities become difficult or impossible at very low electron energies, the observations of electron mobility form a most useful method of studying these properties for electron energies between 0 and 10 volts.

It is the purpose of this paper to describe a new and accurate method for the absolute determination of electron mobilities and to present the initial results on the mobilities of electrons in hydrogen.

EXPERIMENTAL METHOD

The principle of the method lies in the application of the electron-ion filter designed by Loeb and developed by Lusk and Cravath[1,2] to the electrical shutter method of measuring ion mobilities first used by Van de Graaff.[3] A diagram of the apparatus is given in Fig. 1. Electrons which are emitted from the lower zinc plate, P, by ultraviolet light pass upward through the gas under the influence of a uniform field. Between alternate wires of the grids G and G' is applied a high frequency alternating potential whose mean value is the value of the uniform field at that point. The frequency and magnitude of this field may be varied within wide limits. In general, electrons which reach G will be swept out to the grid wires (if the potential between them is high enough) and no current will be transmitted. If, however, the magnitude of the potential between adjacent grid wires is reduced, an intermittent electron current will eventually pass through, consisting, obviously, of those electrons which have reached the grid at a time when the alternating field was zero or nearly so. Under these circumstances a series of pulses of electrons will pass through the grid and be carried by the uniform field towards the grid G'. This grid is subjected to an alternating field of the same frequency, magnitude, and phase as the grid G.

[1] A. M. Cravath, Phys. Rev. **33**, 605 (1929).
[2] N. E. Bradbury, Phys. Rev. **44**, 883 (1933).
[3] R. J. Van de Graaff, Phil. Mag. **6**, 210 (1929).

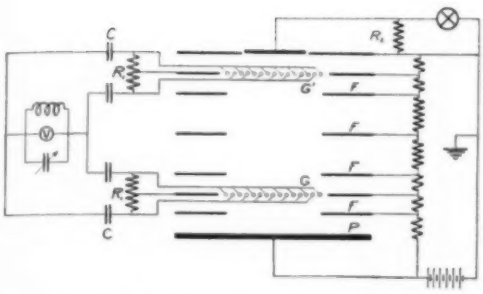

FIG. 1. Schematic diagram of apparatus. $R_1 = 10^6\Omega$ and $R_2 = 10^9\Omega$. $C = 0.01$ mf.

If a given electron pulse reaches this grid at any time other than when the voltage wave is again passing through zero, it will be swept out to the grid wires, and no current will reach the collecting electrode. However, if the drift velocity of the electrons is such that they traverse the distance between the grids and reach G' in exactly one half-cycle (or a multiple thereof) the voltage between the grid wires will be zero, and a current will pass through to the collecting electrode. Thus, varying either the speed of the electrons or the frequency of the alternating potential will give a sharp maximum in the electrometer current at that combination of frequency and speed which permits the electrons to cross the space between the grids in exactly a half-cycle. Since the time for this passage is given directly by the frequency, and the distance between the grids is known, the drift velocity and mobility may be immediately calculated.

In the experimental tube the distance between the grids G and G' was 5.93 cm. The grids themselves were of No. 40 copper wires mounted 1.0 mm apart on mica frames. They were connected as shown in the figure by short symmetrical leads to a tank circuit which was in turn coupled to a radiofrequency oscillator. The frequency of the latter could be varied between 10^4 and 10^7 cycles/sec., and the potential applied to the grids between 0 and 200 volts. Guard rings, $F \cdots F$, maintained a uniform field in the space through which the electrons moved. The apparatus was sealed in a Pyrex tube with a graded quartz seal for the admission of ultraviolet light from a quartz mercury arc. The apparatus could be baked out at a moderate temperature and pumped down prior to the admission of gas to less than 10^{-7} mm Hg.

Since the current which is passed by a grid is a rapidly varying function of the voltage between adjacent grid wires, it was found easier in practice to fix the oscillator at a constant frequency and voltage and vary the speed of the electrons by varying the total potential applied across the tube. This gives a current voltage curve which has a maximum at a value of X/p (field strength in volts per cm and pressure in mm Hg) to which the electron velocity, calculated from the frequency, corresponds. It is necessary to make a slight correction for current magnitude to the experimental readings since the total current leaving the lower plate is a function of the potential across the tube, but this is quickly determined and applied.

EXPERIMENTAL RESULTS

The hydrogen used in these experiments was obtained from a commercial tank of the gas and purified by passing over hot copper gauze and through a series of traps immersed in liquid air. Experiments in which the liquid air was replaced by solid CO_2 gave identical results. No negative ions could be detected at the pressures used (1.5 mm to 50 mm), even after the gas had been in the tube for several days. Frequencies were measured by a vacuum tube wave meter which had been calibrated against broadcast frequencies and their harmonics. The potentials applied to the grid were measured by a Compton electrometer with a phosphor bronze suspension. One pair of quadrants was removed from the instrument which was used idiostatically.

A typical experimental curve is shown in Fig. 2. This was taken at a pressure of 6.8 mm and a frequency of 2.205×10^5 cycles/sec. No special care was taken to obtain sharp peaks in this curve since it was desired to show the three peaks on the same scale. Maxima are observed at voltages of 390, 119, and 62 corresponding to values of X/p of 5.73, 1.75, and 0.91. Peak A is due to electrons which cross in one half-cycle, B in one cycle, and C in three halves of a cycle. The velocities which correspond to these orders are evidently 26.5, 13.2, and 8.84×10^5 cm/sec., respectively.

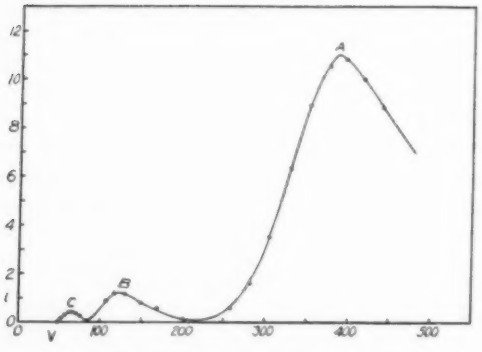

FIG. 2. Typical experimental curve showing peaks due to electrons which cross space between grids in $\frac{1}{2}$, 1, and 3/2 cycle.

Obviously, it is desirable to have the experimental peaks as sharp as possible, and it is also of interest to examine the shape and magnitude of the experimental curves as compared with the theoretical. Accordingly an investigation of these points was carried out. The theoretical efficiency of the grids can be calculated from the electrostatics of such a grating provided the electrons follow the lines of force. At these pressures, however, there is considerable diffusion and in consequence the actual efficiency is less than that calculated in this manner. Hence it was found advisable to determine the grid efficiency experimentally. This was accomplished by applying a fixed d.c. potential between the grid wires in place of the usual alternating potential. The fraction of current transmitted by a single grid was then measured as a function of this voltage. Such curves are shown in Fig. 3A for different electron velocities. With these curves it is easy to calculate the shape of an electron pulse given by a sinusoidal wave form, obtaining the curves shown in Fig. 3B. Finally with *two* grids, the resultant current received by the electrometer as a function of the phase of the arrival of the electron pulse at the second grid may be determined and such curves are exhibited in Fig. 3C. It is seen that the breadth at half-maximum decreases as the grid voltage is increased, and hence the peaks may be made as narrow as the sensitivity of the current-measuring device will permit. In practice a Compton electrometer of sensitivity 10,000 mm/volt gave ample accuracy in determination of the peak.

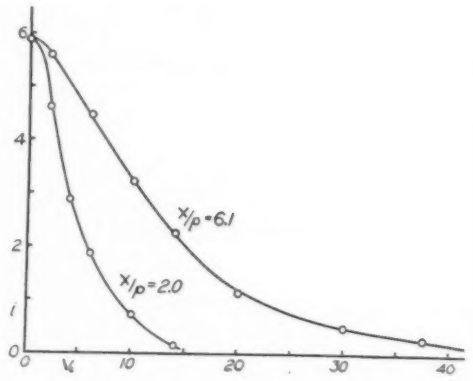

FIG. 3A. Current transmitted by a single grid when a d.c. potential is applied between adjacent grid wires. Pressure = 6.8 mm.

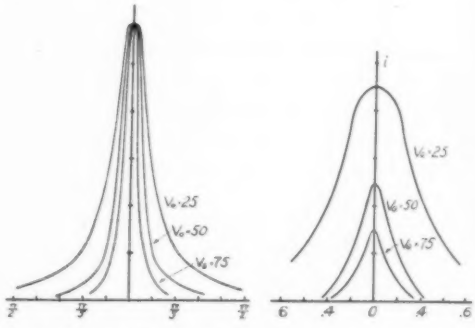

FIG. 3B (left). Form of current pulse transmitted by a single grid for different peak a.c. voltages.
FIG. 3C (right). Calculated form of current peaks received by collecting electrode as a function of phase of arrival of electrons at second grid.

An example of a first-order peak under the conditions given in the caption is shown in Fig. 3D. The calculated curve, now as a function of voltage, is shown in the dotted line. The agreement in form of the two curves is most gratifying. Actually, there should be another small correction applied to the experimental curves to take account of the variation in grid efficiency with the velocity of the electron. This, however, is a very small effect for a peak of ordinary breadth and makes no measurable change in the position of the maximum. In addition the position of the peaks was found to be independent of the grid voltage, and measurements in either the first or second orders, at the same X/p, gave identical values.

A summary of the data taken at a temperature

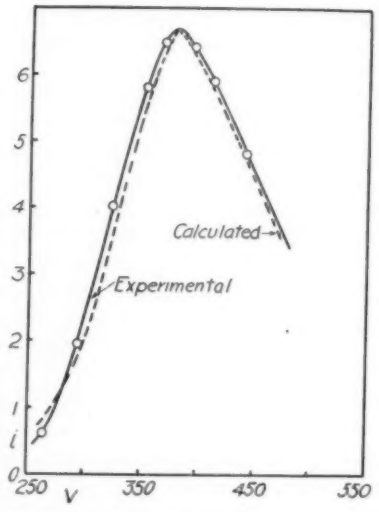

FIG. 3D. Comparison of calculated and observed peaks in first order with a pressure of 6.8 mm and a frequency of 2.2×10^6 cycles/sec.

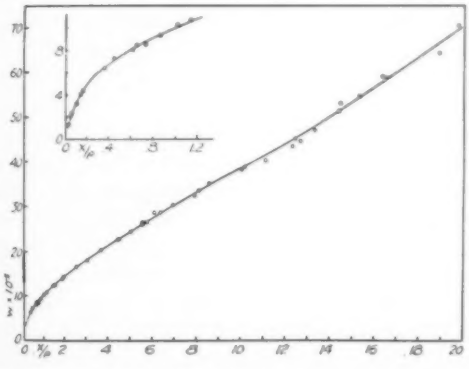

FIG. 4. Drift velocity of electrons in hydrogen as a function of X/p.

of 20°C between values of $X/p=0.03$ and 20 is shown in Fig. 4. The actual drift velocity, w, of the electrons is plotted instead of the mobility, inasmuch as this is the measured and generally more useful quantity. The mobility constant, $K_{20°}$, may be obtained immediately by the equation

$$K=w[(X/p)760]^{-1}. \qquad (1)$$

The agreement of successive experimental points taken at different pressures and in different orders is very good. At the highest values of X/p, the oscillator voltage could not be raised high enough to obtain sharp peaks, and the difficulty of measuring closely pressures of the order of 2 mm cause some of the highest values to be slightly less accurate.

The values of the drift velocity obtained in this experiment lie between those of Townsend and Bailey[4] (magnetic deflection method) and those of Loeb[5] (Rutherford a.c. method). They agree better with the former at high X/p, where a consideration of the magnetic method would lead one to expect the greatest accuracy, and lie fairly close to Loeb's at low X/p. Hence it is believed that the present values represent quite accurately the value of the drift velocity of electrons in hydrogen over the range of X/p studied.

DISCUSSION

A theoretical derivation of an equation for electron mobilities has been given by Compton.[6] In this derivation it is assumed that the collision of an electron and a molecule is that of elastic spheres, and hence that scattering is equally probable in all directions. Furthermore, the mean free path under such a definition has a real meaning and is taken as $(N\pi\sigma^2)^{-1}$ where N is the number of molecules per unit volume, and σ the molecular collision radius. Each collision is then assumed to be completely elastic and the fraction of electron energy transferred from electron to molecule is $4m/M(\sin^2\theta)$, where m and M are the masses of the electron and molecule, and θ is the angle through which the relative velocity of the particles is turned at a collision. Equating the energy gain per mean free path from the field to the energy loss by momentum transfer at collision, an equation for the drift velocity, w, as a function of X/p is obtained which is of the form

$$w=\frac{1.115\times10^{-10}(X/p)(1/\sigma^2)T^{\frac{1}{2}}}{[1+(1+1.265\times10^{-22}(X/p)^2 1/\sigma^4 f)^{\frac{1}{2}}]^{\frac{1}{2}}}, \qquad (2)$$

[4] Townsend and Bailey, Phil. Mag. **47**, 873 (1921).

[5] L. B. Loeb, Phys. Rev. **19**, 24 (1922).

[6] K. T. Compton, Rev. Mod. Phys. **2**, 123 (1930); L. B. Loeb, *Kinetic Theory of Gases* (McGraw-Hill, New York, 1934), p. 600; Morse, Allis and Lamar Phys. Rev. **48**, 412 (1935)) have derived an expression for the electron drift velocity which is essentially the same as that of Compton if the thermal energy of the molecules is neglected. There is a slight (10 percent) difference in the numerical constant. Davydov (Physik. Zeits. Sowjetunion **8**, 59 (1935)) has made similar calculations and reached the same result although his final equation has a numerical error.

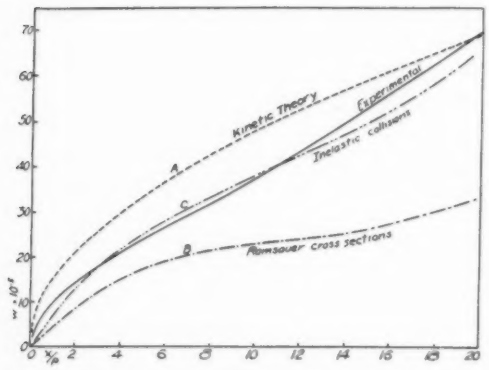

FIG. 5. Comparison of experimental and calculated curves based on the Compton electron mobility equation. Curve *A* is calculated with spherical scattering and cross section equal to the kinetic theory value: Curve *B*, spherical scattering, but Normand's values of the hydrogen molecule cross section; and Curve *C* assumes inelastic collisions with Normand's cross sections.

in which f is the fraction of its energy which an electron loses at a collision.

Assuming for the moment that the collisions are those of elastic spheres, the average energy loss becomes $2m/M$. Taking the kinetic theory value of $\sigma=1.045\times10^{-8}$ cm, and substituting M for H_2, one obtains curve *A* in Fig. 5. It is seen that in general the theory predicts a drift velocity of the proper order of magnitude, but that there are rather serious discrepancies. These pertain to both the character and trend of the curve, and the actual magnitude of the values predicted. However, it is a well-known fact that the scattering cross section of molecules varies greatly with the energy of the colliding electron. Townsend[7] has given some values of the electron energy in hydrogen as a function of X/p which were obtained by measuring the lateral diffusion of a beam of electrons passing through a gas. For the range of X/p in which we are interested, these energies lie between 0 and 3 electron volts; electrons of one electron volt average energy occurring at an X/p of approximately five. With the help of this information, one may calculate w from (2) using the data of Normand[8] on Ramsauer cross sections in H_2. This gives curve *B* of Fig. 6. This again fails to agree in magnitude with the experimental curve, but an improvement is seen in the general trend. This arises from the fact that Ramsauer cross sections in H_2 decrease rapidly with increasing energy, thus increasing w.

The discrepancy in magnitude, which is far greater than can be accounted for by possible experimental inaccuracies in the observation of Ramsauer cross sections, must still be considered. The above calculations have been based on the assumption that the scattering is spherically symmetrical. Actually the experiments of Ramsauer and Kollath[9] have shown that in the case of hydrogen and helium there is a pronounced tendency towards backwards scattering at very low electron energies (0–10 volts). This will tend to increase f since the energy loss depends upon $\sin 2\theta$. No great benefit results, though, even if it is assumed that *all* the electrons are scattered backwards, since this will merely double f and increase w by a factor of approximately $2^{\frac{1}{2}}$. It seems inescapable that the average fraction of its energy which an electron can transmit to a molecule must be greater than that indicated by classical momentum transfer. In diatomic molecules, the comparatively close spacing of rotational and vibrational levels make such transfers energetically possible, and it has been suggested that such energy losses do occur, even when the colliding electron does not have sufficient energy to raise the molecule to a higher electronic state. The mechanism accomplishing this must have its origin in a perturbation of the nuclear binding forces by the presence of the slow colliding electron resulting in the possible excitation of a vibrational level according to the Franck-Condon principle. Such inelastic impacts have not infrequently been observed.[10] Ramien[11] has investigated the energy loss of slow electrons in hydrogen and found that on the average a 4.15-volt electron loses 0.02 volt energy per collision. Hence it seems most likely that the main cause of the lack of agreement between the experimental and the theoretical curve lies in the choice of the factor f. If it be assumed that the factor f is sixteen times the classical elastic value at $X/p=20$ (a not unreasonable value in view of Ramien's results) and also that

[7] J. S. Townsend, J. Frank. Inst. **200**, 563 (1925).
[8] C. E. Normand, Phys. Rev. **35**, 1217 (1930).
[9] Ramsauer and Kollath, Ann. d. Physik **12**, 524 (1932).
[10] H. Baerwald, Ann. d. Physik **76**, 829 (1925); W. Harries, Zeits. f. Physik **42**, 26 (1927); also Loeb, reference 5, p. 603.
[11] H. Ramien, Zeits. f. Physik **70**, 353 (1931).

f increases linearly with electron energy, one obtains the surprisingly good curve C shown in Fig. 6. This agreement is probably partially fortuitous, and a more rigorous development must eventually involve allowances for the effects of varying cross section, scattering angle, and inelasticity of impact in detail. The general character of the problem, however, seems to be indicated by the above considerations, and furnishes further evidence for the inelasticity of electron impact in molecular gases.

These experiments are being continued to determine electron mobilities in other gases. The authors desire to thank the Research Committee of Stanford University for a grant which has made these investigations possible.

MARCH 1, 1936 PHYSICAL REVIEW VOLUME 49

Note on the Quantum-Mechanical Theory of Measurement (see also page 476)

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(Received November 12, 1935)

In recent notes by Einstein, Podolsky and Rosen and by Bohr, attention has been called to the fact that certain results of quantum mechanics are not to be reconciled with the assumption that a system has independently real properties as soon as it is free from mechanical interference. We here investigate in general, and in abstract terms, the extent of this disagreement. When suitably formulated, such an assumption gives to certain types of questions the same answers as does quantum mechanics; this is true of the formulas usually given in discussions of the theory of measurement. There exists, however, a general class of cases in which contradictions occur. That such contradictions are not restricted to the abstract mathematical theory, but can be realized in the commonest physical terms, is shown by the working out of an example.

1. INTRODUCTION

SOME time ago there appeared a paper by Einstein, Podolsky and Rosen[1] entitled "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" The writers concluded that the answer must be negative, on the ground that quantum mechanics forbids simultaneous measurement of two noncommuting variables even when both variables simultaneously possess "physical reality," in the sense that either might be measured "without in any way disturbing the system." Recently Bohr[2] has upheld the view that quantum-mechanical description of nature can be considered complete, by demonstrating how the restrictions on simultaneous measurement which it imposes are inherent in the character and use of the measuring instruments. These measuring instruments must always be included as part of the physical situation from which our experience is obtained, and by doing this one sees that quantum mechanics provides a complete and peculiarly apt interpretation of experience.

Bohr has again clearly called attention to this circumstance, and has remarked that one must be careful not to suppose that a system is an independent seat of "real" attributes simply because it has ceased to interact dynamically with other systems. The paper of Einstein, Podolsky and Rosen has shown the sort of situations in which this characteristic of quantum mechanics may become especially prominent. This indicates an extension of the usual discussions of the theory of measurement.[3] In the present note a discussion more comprehensive in this respect will be summarized, and some further consideration will be given to the possibility of illustrating the point in question in concrete physical terms.

We shall have to make use of the concepts and results presented in von Neumann's rigorous and

[1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935). Referred to as EPR.

[2] N. Bohr, Phys. Rev. **48**, 696 (1935).

[3] Cf. W. Heisenberg, *The Physical Principles of the Quantum Theory*, particularly pp. 55ff.; J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Chap. VI; W. Pauli, *Handbuch der Physik*, Vol. 24, No. 1, pp. 143ff., p. 165.

detailed discussion[3] of the theory of measurement by means of an instrument. Since the mathematical language of von Neumann's work is not that most current among physicists, it is desirable first to explain the meaning of these concepts in more usual terms. A similar rephrasing of the proofs of important results is omitted in the interests of brevity, and the results are simply stated.

2. Possible Types of Statistical Information about a System

Our statistical information about a system may always be expressed by giving the expectation values of all observables.[4] Now the expectation value of an arbitrary observable F, for a state whose wave function is φ, is

$$\bar{F}=(\varphi, F\varphi). \tag{1}$$

If we do not know the state of the system, but know that w_i (with $\sum_i w_i=1$) are the respective probabilities of its being in states whose wave functions are φ_i, then we must assign as the expectation value of F the weighted average of its expectation values for the states[5] φ_i. Thus

$$\bar{F}=\sum_i w_i(\varphi_i, F\varphi_i). \tag{2}$$

This formula for $\bar{F}$ is the appropriate one when our system is one of an ensemble[6] of systems of which numbers proportional to w_i are in the states φ_i. It must not be confused with any such formula as

$$\bar{F}=(\sum_i (w_i)^{\frac{1}{2}}\varphi_i,\ F\sum_i (w_i)^{\frac{1}{2}}\varphi_i),$$

which corresponds to the system's having a wave function which is a linear combination of the φ_i. This last formula is of the type of (1), while (2) is an altogether different type.

An alternative way of expressing our statistical information is to give the probability that measurement of an arbitrary observable F will give as result an arbitrary one of its eigenvalues, say δ. When the system is in the state φ, this probability is

$$|(\varphi, \chi_\delta)|^2, \tag{1'}$$

where χ_δ is the eigenfunction of F corresponding to the eigenvalue δ. When we know only that w_i are the probabilities of the system's being in the states φ_i, the probability in question is

$$\sum_i w_i|(\varphi_i, \chi_\delta)|^2. \tag{2'}$$

Formula (2′) is not the same as any special case of (1′) such as

$$|(\sum_i (w_i)^{\frac{1}{2}}\varphi_i, \chi_\delta)|^2.$$

It differs generically from (1′) as (2) does from (1).

When such equations as (1), (1′) hold, we say that the system is in the "pure state" whose wave function is φ. The situation represented by Eqs. (2), (2′) is called a "mixture" of the states φ_i with the weights w_i. It can be shown[7] that the most general type of statistical information about a system is represented by a mixture. A pure state is a special case, with only one nonvanishing w_i. The term "mixture" is usually reserved for cases in which there is more than one nonvanishing w_i. It must again be emphasized that a mixture in this sense is essentially different from any pure state whatever.

3. Reduction of Wave Packets

Let $\Psi(x_1, x_2)$ be the wave function for two systems I and II which have at some previous time interacted and have now ceased to interact. One can show[8] that there always exists an expansion, which is in general unique, in the form

$$\Psi(x_1, x_2)=\sum_k (w_k)^{\frac{1}{2}}\varphi_{\lambda_k}(x_1)\xi_{\rho_k}(x_2), \tag{3}$$

where the φ_{λ_k} are eigenfunctions of an observable L corresponding to eigenvalues λ_k, and the ξ_{ρ_k} are eigenfunctions of an observable R corre-

[4] v. Neumann, reference 3, p. 163. We confine ourselves throughout to the case of discrete spectra. (φ, ψ) means the Hermitian inner product, $\int \varphi^*\psi d\tau$. The same letters, $A, B \cdots$, are used to denote observables and the corresponding operators.

[5] For convenience we shall often refer to "the state ψ" instead of "the state whose wave function is ψ." All functions are normalized unless otherwise stated.

[6] The usefulness of this concept has recently been remarked upon by Kemble (Phys. Rev. **47**, 973 (1935)).

[7] v. Neumann, reference 3, pp. 167–168.

[8] v. Neumann, reference 3, pp. 225 ff. In the following arguments we use the word "observable" to mean "complete set of commuting observables," in the sense of Dirac, *Principles of Quantum Mechanics*, §17 (1st edition). In like fashion a set of eigenvalues of such a set of observables is referred to as an "eigenvalue" of the "observable."

sponding to eigenvalues ρ_k. The λ_k are all distinct, and so are the ρ_k. It can be shown[9] that, so far as system II alone is concerned, the statistical information available when (3) is the wave function of the combined systems is represented by a mixture of the states ξ_{ρ_k} with the weights w_k. A similar result holds, of course, for system I. A measurement of L and R on the total system can never give any other value than ρ_i for R to correspond to the value λ_i of L. Thus a measurement of L suffices to predict the value of R and the state of system II, which, *after*[9] such a measurement, is always in one of the pure states ξ_{ρ_k}. Eq. (3) shows that the coupling between the systems has been such as to make system I a suitable instrument for measuring the observable R on system II, the quantity L serving as a "pointer reading."

Now the conclusions to be drawn on the basis of these developments are just those we should expect if we ascribed "real" characteristics to system II as soon as it ceased to interact with system I; this will be shown explicitly later. The contradictions we wish to investigate can be brought out only by going beyond the considerations given in connection with Eq. (3). We may either look for particular cases in which the expansion (3) is not unique—e.g., the example given by EPR and the one we shall give in Section 5—or develop a way of interpreting expansions of a less special type.

The second alternative brings us directly to the general method of "reducing the wave packet." This procedure is commonly known and accepted among physicists, and is applied by EPR; but the writer has been unable to find in the literature an explicit description of its application to the present case. Such a description is briefly as follows: If M is any observable of system I, and ψ_μ its eigenfunction corresponding to the eigenvalue μ, then we can express $\Psi(x_1, x_2)$ as a series in the orthogonal functions $\psi_\mu(x_1)$, with coefficients which are functions[10] of x_2:

$$\Psi(x_1, x_2) = \sum_\mu \psi_\mu(x_1)\zeta_\mu(x_2), \tag{4}$$

where $$\zeta_{\mu'}(x_2) = \int \psi_{\mu'}^*(x_1)\Psi(x_1, x_2)dx_1. \tag{5}$$

The statistical information which one will have about system II after[9] a measurement of M on system I has given the value μ' may be obtained by the following process: We suppose a large number of measurements made on combined systems I+II prepared so that their wave functions are given by (4), each of these measurements consisting in a determination of the values of M for system I and some observable F for system II. We then obtain the relative numbers of times the different values δ are found for F, counting only those measurements in which the value μ' is found for M. These relative numbers are by definition proportional to the quantities $|(\Psi(x_1, x_2), \psi_{\mu'}(x_1)\chi_\delta(x_2))|^2$ (cf. Eq. (1′)), and these quantities are, by (5), just equal to $|(\zeta_{\mu'}, \chi_\delta)|^2$. Since this is true for all observables F, we see that after a measurement on system I has given the value μ' for M, system II is in the pure state with wave function given—apart from normalization—by (5).

4. Probability Calculations and Their Results

We are now ready to discuss in detail the degrees of agreement and disagreement between the results of quantum-mechanical calculations and those to be expected on the assumption that a system once freed from dynamical interference can be regarded as possessing independently real properties. For we can give a definite form to this assumption, and base on it a method for answering all questions which can be asked about the probabilities of finding different results by measurements on system II. This we call Method A:

Assumption and method A. We assume that during the interaction of the two systems each system made a transition to a definite state, in which it now is, system I being in one of the states φ_{λ_k} and system II in one of the states ξ_{ρ_k}. These transitions are not causally determined, and there is no way of finding out *which* transitions occurred, except by making a suitable measurement. In the absence of measurements we know

[9] i.e., *immediately* after. The wave functions used are in general not stationary solutions of the wave equation, but in our discussion we can abstract from the time, because our statistical information about a system at one time can be calculated from that at another time according to a definite differential equation (v. Neumann, reference 3, p. 186).

[10] Unnormalized, and in general not orthogonal to each other.

only that the probabilities of the different transitions are respectively w_k, and that if system I is in the state φ_{λ_i} system II is in the state ξ_{ρ_i}. This provides a sufficient basis for making all needed calculations of probabilities, the methods being those of ordinary probability theory.

Method B. We shall compare with the results of Method A those obtained by quantum-mechanical calculations, using the facts explained in connection with Eqs. (3) and (4), (5).

There are four types of questions for which answers may be required. The notation used in discussing them is the same as that previously described; in particular, the reader is reminded that the observables L and R have a special significance through their connection with the expansion (3), while M and S are arbitrary observables. The questions and their answers are as follows:

(a) If S, having eigenvalues σ and eigenfunctions η_σ is measured on system II without any measurements having been made on system I, what is the probability of obtaining the result σ'? Both methods give the same result

$$\sum_k w_k |(\xi_{\rho k}, \eta_{\sigma'})|^2, \tag{6}$$

as is at once evident. (Cf. remarks following Eq. (3).)

(b) If L has been measured on system I and the value λ_i obtained, what is the probability of finding the value σ' for S in II? Both methods at once give the answer

$$|(\xi_{\rho i}, \eta_{\sigma'})|^2. \tag{7}$$

When $S=R$ and $\sigma'=\rho'$, we get in particular the value

$$|(\xi_{\rho i}, \xi_{\rho'})|^2 = \delta_{\rho i \rho'}, \tag{7'}$$

so that a definite result is predicted. The possibility of such definite predictions was taken by EPR as a "criterion of the physical reality" of the observable R; it is, *par excellence*, the bit of evidence which might incline one to believe Assumption A to be true.

(c) If M has been measured on system I and the value μ' obtained, what is the probability of finding the value ρ_i for R in II? Some calculations are required for this case, but are omitted, being analogous to those which will be given in detail for (d). Both methods give the result

$$w_i |(\varphi_{\lambda_i}, \psi_{\mu'})|^2 / [\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2]. \tag{8}$$

(d) If M has been measured on system I and the value μ' obtained, what is the probability of finding the value σ' for S in II?

Method A: If the measurements are carried out on a large number of similarly prepared pairs of systems, the fraction giving the value μ' for M is $\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2$. The fraction giving this value and having system II in state $\xi_{\rho k}$ is $w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2$. Then the fraction giving the values μ' for M and σ' for S is $\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2 \times |(\xi_{\rho k}, \eta_{\sigma'})|^2$. Dividing this by the fraction giving the value μ' for M, we find as the required *a posteriori* probability,

$$[\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2 |(\xi_{\rho k}, \eta_{\sigma'})|^2] / [\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2]. \tag{9A}$$

Method B: The wave function from which we must calculate this probability is, by (5), (3):

$$\int \psi_{\mu'}^*(x_1) \Psi(x_1, x_2) dx_1 = \sum_k (w_k)^{\frac{1}{2}} (\psi_{\mu'}, \varphi_{\lambda_k}) \xi_{\rho k}(x_2).$$

On normalizing this function and taking the square of its inner product with $\eta_{\sigma'}$ one gets for the required probability

$$[|\sum_k (w_k)^{\frac{1}{2}} (\varphi_{\lambda_k}, \psi_{\mu'}) (\xi_{\rho k}, \eta_{\sigma'})|^2] / [\sum_k w_k |(\varphi_{\lambda_k}, \psi_{\mu'})|^2], \tag{9B}$$

where the denominator comes from normalization.

The difference between (9A) and (9B) comes from the well-known phenomenon of "interference" between probability amplitudes. The absence of such an effect in case (a) is usually stressed in discussions of the theory, since it shows plainly the effect which the mere attaching of an instrument must in general have on the behavior of a system. Since case (d) is not mentioned, it is possible for a reader to form the

impression that the theory is consistent with Assumption A.[11]

The formal discrepancy between (9A) and (9B) is a consequence of the fact that, according to the remarks following (4) and (5), after a measurement of M on system I has been made system II is in a pure state, which is in general not one of the $\xi_{\rho k}$. Now no possible manipulation of the w_k will produce from the statistics of the mixture those of any pure state other than one of the $\xi_{\rho k}$. Thus not only is Method A inconsistent with Method B, but also there is no conceivable modification of Method A which could produce consistency between Assumption A and Method B.

The contradiction here, like that between quantum mechanics and the classical doctrine of causality, indicates a radical change in concept rather than a mere change in the details of a mechanism. The idea which is found to be untenable may, roughly, be said to be that of the independent existence of two entities, the state of system II and one's knowledge of its state, only the latter being affected by measurements made on system I. Quantum theory shows that this is not an adequate concept of the relation between subject and object.

5. A Physical Example

The inconsistency of quantum mechanics with the point of view which finds its definite formulation in Assumption A has been demonstrated mathematically. One may still wish to inquire whether it can be realized in a concrete physical example; for we may certainly suppose that not all mathematical operators, even though subject to the proper formal requirements,[12] correspond to experimentally measurable quantities.

An example has been outlined in mathematical form by EPR. As Bohr has remarked, and as is evident from the mathematics, the physical realization of this example involves certain difficulties, in particular the necessity of abstracting altogether from the time in circumstances in which this is not really permissible. We shall now outline an example in physical terms, in which full account is taken of the time-dependence of the quantities involved.

In order to give a physical example of the point in question, it would suffice to describe any case in which one system (I) is used as a measuring instrument in observing another system (II), and in which, after interaction has ceased, some other observable besides the one suited to serve as a "pointer reading" can be measured on system I. The resulting inferences about system II would in the main fall under case (d), in which the contradiction between quantum mechanics and Assumption A is evident. A variety of such examples could doubtless be given.[13] There is, however, a particularly neat and striking special type of example, to which EPR have directed attention; and the one we shall describe is of this type. Before describing it in detail, we shall indicate the nature of this sort of example and its connection with the argument of the preceding section.

The characteristic feature of such an example is obtained by choosing a case in which the expansion (3) is not unique.[14] An assumption in the form of Assumption A can be stated corresponding to each of two expansions of type (3). Each of these assumptions is consistent with a number of quantum-mechanical results of the forms (6), (7), (8), and particularly of the form (7′); thus both of them are to be accepted as

[11] Cf. Pauli, reference 3, p. 89. The remarks there given are entirely correct, but liable to be misleading to an unwary reader with a predilection for Assumption A. The same is to some extent true of the remarks in v. Neumann, reference 3, p. 232, and Heisenberg, reference 3, pp. 59–62.

[12] v. Neumann, reference 3, pp. 75 ff.

[13] e.g., the example we shall discuss could be stated in this way. In the construction and discussion of such examples a difficulty arises owing to the fact that in experimental practice the observables used are almost always incomplete (cf. note 8), and that the theory of measurements of incomplete observables is not altogether free from ambiguity (cf. v. Neumann, reference 3, pp. 184–185). For this reason it is much easier and more satisfactory to treat our example—which itself involves the use of incomplete observables—in the way we have here used.

[14] The condition for this is that the w_k be not all distinct (cf. v. Neumann, reference 3, p. 232 and p. 175). In the example of EPR, the w_k are all equal and *all* states of system II are included. This means that there are an infinite number of different expansions of the form (3), and that any measurement made is as likely to give any one result as any other. Measurement in quantum mechanics has in general a twofold aspect: it gives information (of a statistical nature) about the properties of the state of the system *before* the measurement, and it enables us to predict the state of the system *after* the measurement. In order that it may serve the first purpose, care must be used in choosing a suitable coupling of object and instrument. In the example of EPR, the coupling has been so violent that all trace of the original state of system II is lost, so that the word "prediction" is the only correct one to apply to one's conclusions about system II.

true, according to the general attitude underlying Assumption A. Now each assumption asserts that system II has made a transition to a definite (though unknown) state, and that it is a state for which a certain observable has a well-defined value. In the case chosen for the example the two requirements thus imposed on the state of system II cannot be fulfilled simultaneously by any quantum-mechanical state, because of the limitations imposed by the uncertainty principle. The point of view expressed in Assumption A is accordingly found to conflict with quantum mechanics.

With this outline of the argument in mind, we proceed to the detailed discussion of the example. In this we shall use the language of experimental physics rather than that of the mathematical formalism.

We consider the determination of the position of a heavy particle of mass M, say a proton, by the use of a "microscope." This microscope is designed for use, not with γ-rays, but with light charged particles of mass m, say electrons. If we shine soft light in from the side of the "barrel" and let it be scattered from an electron which is on its way to the "lens," a transverse component[15] of the electron's momentum can be calculated from the Doppler effect on the scattered light. Under suitable circumstances this will enable us to infer the momentum[15] of the proton quite accurately. On the other hand, we have only to let the electron travel undisturbed to the photographic plate in order to be able to infer with considerable accuracy the corresponding coordinate[15] of the proton. The "Assumption A" which corresponds to the first experiment is: During the collision between the electron and the proton, the proton made a transition into some state with well-defined momentum; *which* state this is can be determined by a measurement made on the electron. Corresponding to the second experiment one has an "Assumption A" which reads exactly the same except that the word "momentum" is replaced by the word "coordinate." Since one is still free to choose which experiment is to be performed *after* the electron has been scattered into the microscope[16] and ceased to interact with the proton, these assumptions must both be true at once, if one accepts the point of view on which they are based. But we shall see that their simultaneous truth can be in conflict with the uncertainty principle.

Before deriving the actual expressions for the uncertainties in the two alternative predictions, let us consider briefly their physical origin. In principle the Doppler effect experiment can be made with arbitrary precision (cf. latter part of note 16). The uncertainty Δp may accordingly be regarded as fixed by the original uncertainties in the momenta of the particles; to make it small, we have only to prepare them properly beforehand. Now the use of particles so prepared will in some degree limit the accuracy of our prediction of x. This comes about through the fact that the predictions we want must refer to a definite time, and that the prediction of x read directly from the photographic plate refers to the moment of the collision: this is not precisely known, because our long wave trains take a finite time to pass over each other. In making a prediction for a definite moment, which we choose to be that at which the wave trains cease to overlap, we must allow for the distance the proton may have moved since the collision. This leads to the existence of a lower limit for the product $\Delta p \Delta x$; but since the electron, with its small mass, moves rapidly across the region where the proton may be found, whereas the proton's motion after the collision is comparatively sluggish, this limit will be found to be not h, but about $(m/M)h$.

[15] i.e., transverse to the axis of the microscope. This component of the momentum will be called simply the "momentum," and the corresponding coordinate will be called the "coordinate" x.

[16] Of course the electron need not have been scattered into the microscope; in any given case we can only go on with the experiment in good faith, hoping that it will have been so scattered. The experiment can fail; and from the fact that it has succeeded when one choice was made, quantum mechanics of course offers no such inference as that it "would have" succeeded if the other choice had been made. But under the proper circumstances, such an inference does follow from the point of view of Assumption A. The circumstances required are, that in the Doppler effect experiment enough quanta must be sent in so that "if the electron is in the barrel," several are sure to be scattered back into a suitably small solid angle. The quanta must be extremely soft, so that scattering a great many of them will not change the electron's momentum too much; and their energy must be measured very accurately. This means that the electron is in the barrel a long time; and it turns out that, to secure the accuracy represented by Eq. (14), the length of the microscope must be much greater than $(M/m)^{11/3}\Delta x$, while the lateral distance to the source and analyzer of the light must be much greater than $(M/m)^{13/3}\Delta x$. These requirements make in principle no difficulty.

To show this, we shall discuss the possible sources of uncertainty in the prediction of x. These are: (1) The finite resolving power of the microscope; (2) inaccuracy of focus; (3) allowance for the proton's motion between the moment of the collision and the moment to which the predictions refer.

(*1*) *Finite resolving power.* This gives an uncertainty

$$\Delta_1 x \sim (\lambda/\epsilon) \sim (h/p\epsilon), \tag{10}$$

where ϵ is the numerical aperture and λ, p are wave-length and momentum of the electron.

(*2*) *Inaccuracy of focus.* Let the electron[17] be sent in through a slit of width s. At the far side of the field of view the half-width of the beam will have become, through diffraction, about $(s/2)+(L\lambda/2s)$, where L is the breadth of the field of view. By proper choice of s, this expression takes on its minimum value, $(\lambda L)^{\frac{1}{2}}$. Now L must be equal to $(h/\Delta p)$, the whole length of the wave train; otherwise the restriction of the field of view would cause enough diffraction of the scattered waves to spoil the significance of the contemplated Doppler effect experiment. Thus we get for the uncertainty in x from this source

$$\Delta_2 x \sim \epsilon(\lambda h/\Delta p)^{\frac{1}{2}} = \epsilon h/(p\Delta p)^{\frac{1}{2}}. \tag{11}$$

(*3*) *Allowance for proton's motion.* The time available for this motion is of the order of magnitude

$$\Delta t \sim (h/\Delta p)/(p/m),$$

which is the time required for the electron to travel the length of such a wave train as must be used. To make the proton's velocity after the collision as small as possible, we can send the two particles into the field of view with equal and opposite momenta. Then if the electron were scattered exactly at a right angle, the resulting x component of the proton's velocity would be zero. But on account of the finite aperture we must take it to be roughly

$$v_x' \sim \epsilon(p/M).$$

We then get

$$\Delta_3 x \sim v_x' \Delta t \sim \epsilon(m/M)(h/\Delta p). \tag{12}$$

Thus we have finally

$$\Delta p \Delta x \sim \Delta p(\Delta_1 x + \Delta_2 x + \Delta_3 x)$$
$$\sim h\{(\Delta p/\epsilon p) + \epsilon(\Delta p/p)^{\frac{1}{2}} + \epsilon(m/M)\}. \tag{13}$$

By making Δp extremely small compared to p, we can make

$$\Delta p \Delta x \sim (m/M)h. \tag{14}$$

By taking ϵ also to be small, we can in principle make $\Delta p \Delta x$ *arbitrarily* small; also we could in principle dispense with the advantage we obtained by using the disparity in mass of two known particles.

As explained in the preliminary discussion of the example, the comparison of a result such as (14) with the uncertainty principle shows that Assumption A is inconsistent with quantum mechanics.

Concluding Remark

Both by mathematical arguments and by discussion of a conceptual experiment we have seen that the assumption that a system when free from mechanical interference necessarily has independently real properties is contradicted by quantum mechanics. This conclusion means that a system and the means used to observe it are to be regarded as related in a more subtle and intimate way than was assumed in classical theory. It does not mean that quantum mechanics is not to be regarded as a satisfactory way of correlating and describing experience; it does illustrate the difficulty, often remarked upon by Bohr, which is inherent in the problem of the distinction between subject and object.

[17] Only one beam need be narrow. Under (3) we shall see that it is expedient to make the wave-lengths of electron and proton originally equal, so that $\Delta_2 x$ will be the same whichever beam is limited. By admitting the proton through a slit much broader than s, we can assure that the electron escapes unless it is scattered through a fairly large angle, thus avoiding the prevalence of spurious effects.

LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Communications should not in general exceed 600 words in length.

Continuation of Work on Scattering of Slow Neutrons

Some time ago the authors[1] published a report on the scattering of slow neutrons by various metals using a thin silver sheet as a detector. The experiments have been extended to include a number of other materials as scatterers so that data now exist on the scattering cross section for some sixteen elements distributed throughout the periodic table.

The experimental arrangement in all of the experiments described below was the same as that previously used. Neutrons, from a radon-beryllium source, which had passed through 6 cm of paraffin, struck a silver foil 6×10 cm and then were scattered from blocks of metal the same size as the silver foil, placed above it. The scattering was measured by observing the increase in the radioactivity of the silver foil caused by the presence of various thicknesses of scatterer. In the case of Hg the liquid, and in that of Mn and Cr the powdered, elements were used. These were contained in a box, the scattering from which was accounted for. The results of a typical experiment are shown in Fig. 1 in which the percentage scattering is plotted as ordinate against the thickness of scatterer as abscissa. The curves represent the scattering of Ni, C, Ag and Al with a silver detector.

It will be seen from the figure that Ni and C are good scatterers whereas Ag and Al are not. Relative scattering cross sections may be obtained by placing the slope of the curve, in its initial linear portion, equal to $N\sigma^2$, where N is the number of atoms per cc used, σ^2 the relative scattering cross section. The results for all elements so far investigated

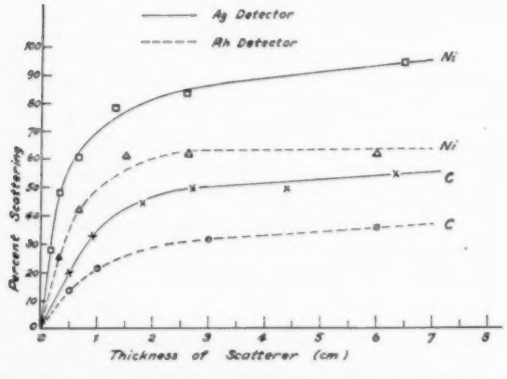

FIG. 1. Percent scattering of slow neutrons from Ni, C, Ag and Al (Ag detector).

TABLE I. *Percent scattering of slow neutrons of various elements.*

ELEMENT	AT. NO.	$\sigma^2\times10^{24}$cm² SCATTERING	$\sigma^2\times10^{24}$cm² D, P. F & M	MAX. PERCENT SCATTERING
C	6	3.2	4.1	57
Mg	12	2.8	3.5	(34 at 6 cm)
Al	13	0.9	1.5	(20 at 6 cm)
S	16	0.8	1.4	(17 at 6 cm)
Cr	24	1.3	4.9	12
Mn	25	2.0	14.3	16
Fe	26	9.9	12.0	88
Ni	28	17	15.4	95
Cu	29	7.7	7.5	55
Zn	30	3.4	4.7	37
Ag	47	5.9	55	8
Cd	48	(1.2)	3300	(2.4 at 4 mm)
Sn	50	3.8	4.0	32
Hg	80	4.4	380	20
Pb	82	7.2	8.6	52
Bi	83	9.5	8.2	40

are shown in Table I. In column 3 of the table is given the value of σ^2 observed in these experiments; in column 4 the values for the "absorption" cross section observed by Dunning, Pegram, Fink and Mitchell;[2] and in column 5 the maximum percentage scattering observed by the present authors.

It should be noted that in all cases except Ag, Hg, and Cd the scattering cross section is about the same as that measured by the "absorption" method. The metals Ag, Hg, and Cd are known to have large capture cross sections for slow neutrons. Measurements made by the "absorption" method give the sum of capture cross section plus scattering cross sections. It will be seen from the table that the scattering cross section shown by these elements is of the same order of magnitude as that shown by the other elements investigated and is not anamolously large as is the capture cross section. Furthermore, the scattering cross sections for the various elements, with the exception of Ag, Hg, Cd and possibly Mn, are approximately the same as the "absorption" coefficients observed by Dunning *et al.* This gives additional confirmation to the idea that the observed "absorption" of most of the metals investigated is largely due to scattering.

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LAWRENCE M. LANGER

New York University,
University Heights,
February 10, 1936.

[1] A. C. G. Mitchell and E. J. Murphy, Phys. Rev. **48**, 653 (1935); **47**, 881 (1935).
[2] J. R. Dunning, G. B. Pegram, G. A. Fink and D. P. Mitchell, Phys. Rev. **48**, 265 (1935).

Selective Scattering of Slow Neutrons

In addition to the experiments described in the foregoing letter we have investigated the scattering of slow neutrons from various scatterers using In and Rh detectors. The Rh detector used was 0.1 mm thick (0.122 g/cm²), the In detector 0.15 mm (0.109 g/cm²), while the Ag detector was 0.1 mm (0.105 g/cm²).

Complete scattering curves were run, with a Rh detector, for the scatterers Ni, C, and Zn while a sufficient number of points was observed on the curve for Fe to show the trend of the curve. Typical results are shown in Fig. 1 where the full curves give the scattering with Ag detector and the dashed curves that for the Rh detector. In addition, single points were obtained with the Rh detector for the metals Pb, Cu, Al and Bi. Because of the long period of In no curves were run with this detector, but points were taken showing the scattering for the metals Fe, Ni, Pb, and Cu. In Table I are shown the relative cross section for scattering for the Ag and Rh detectors for scatterers for which these data are available. Table II gives the comparative scattering (in percent) for the various detectors at specified thickness of the various scattering materials. It will be seen that large differences exist between the measured scattering from a single element by using different detectors. Similar results have been obtained by Tillman[1] on the scattering of slow neutrons by using Cu, Ag, and I, as detectors. Furthermore, the absorption coefficient of various metals has been found by several authors[2] to depend on the detector. In discussing the results it should be noted that each detector contained approximately the same number of g/cm² and, on account of the closeness in atomic weights of the three metals, about the same number of atoms cm/² (to within 15 percent). Let us consider Ag and Rh, since more data exist for these elements. Several explanations are possible. (1) Neutrons which are absorbed by Rh are not scattered back by the various scatterers in such great numbers as those absorbed by Ag. (2) The absorption *vs.* velocity curve may be of such a form that in the region of a most probable velocity for absorption it is steeper and narrower in the case of Rh than in the case of Ag, so that a larger proportion of neutrons having the necessary energy to activate Rh strongly are absorbed in the first passage through the metal than in the case of silver. Probably some combination of the two explanations is actually the real one. In favor of (1) the authors have shown[3] that, by using an Ag detector and filtering the neutrons through 0.65 mm Cd, a smaller cross section for the scattering by ions was obtained than with unfiltered neutrons.

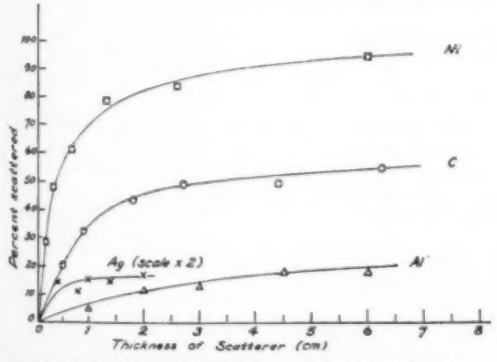

FIG. 1. Percent scattering of slow neutrons from Ni and C.

TABLE I. *Relative scattering cross sections, Ag and Rh detectors.*

SCATTERER	$\sigma^2 \times 10^{26}$cm² Ag DETECTOR	$\sigma^2 \times 10^{26}$cm² Rh DETECTOR
C	3.2	2.2
Fe	9.9	4.3
Ni	17	7.2
Zn	3.4	2.9

TABLE II. *Comparison of various detectors (percent scattering).*

SCATTERER DETECTOR	1 cm Fe	0.65 cm Ni	1 cm Pb	1.3 cm Cu	2.1 cm Sn	5 cm Al	2 cm Bi	1 cm C	1 cm Zn
Ag	59	60.5	25.0	45.0	22.0	18.0	35	35.0	20.0
Rh	32.8	42.0	26.8	—	19.6	18.7	26.1	21.0	19.0
In	51.0	51.0	18.2	40.2	—	—	—	—	—

We are indebted to Dr. C. B. Braestrup of the Physical Laboratory of the Department of Hospitals of the City of New York for many favors and also to the American Association for the Advancement of Science for a grant to one of us (A. C. G. M.) with the help of which apparatus has been purchased.

ALLAN C. G. MITCHELL
EDGAR J. MURPHY
MARTIN D. WHITAKER

New York University,
University Heights,
February 10, 1936.

[1] J. R. Tillman, Nature **137**, 107 (1936).
[2] Moon and Tillman, Nature **135**, 904 (1935); Tillman and Moon, Nature **136**, 66 (1935); Ridinour and Yost, Phys. Rev. **48**, 383 (1935); Szillard, Nature, **136**, 950 (1935).
[3] A. C. G. Mitchell and E. J. Murphy, Phys. Rev. **48**, 653 (1935).

Disintegration of the Deuteron by Gamma-Rays

The cross section for photoelectric disintegration of a deuteron by gamma-rays on the assumption of a short range interaction force between neutron and proton, was found by Bethe and Peierls,[1] to be of the order of 7×10^{-28} cm² for $h\nu = 2.62$ MEV. This appears to be in satisfactory agreement with the experimental result 5×10^{-28} cm² of Chadwick and Goldhaber,[2] who allow a factor two for experimental uncertainties.

However, it now seems possible with this experimental accuracy, or with accuracy only slightly improved over this, to set an upper limit on the range of the proton-neutron interaction. The argument is something as follows:

The Bethe-Peierls result for the cross section σ may be considered an exact evaluation if the interaction occurs only over a distance r_0, where r_0 tends to the limit zero. But if r_0 is in the neighborhood of 1.5×10^{-13} cm, in accordance with the idea of various writers, changes occur which tend to increase σ substantially over the value 7×10^{-28} cm². These changes were studied for several types of n–p interaction, including a potential hole of width r_0 and depth V_0, and the exponential function $V = -V_0 \exp(-r/r_0)$. In all cases the

effect was a fractional increase in σ of amount somewhat greater than $r_0E^{\frac{1}{2}}$, where E is the mass defect of deuterium in MEV (about 2.14 MEV), and r_0 is the interaction distance in units of 6.35×10^{-13} cm ($r_0=0.237$ for an interaction extending to 1.5×10^{-13} cm). For the potential hole, and an r_0 corresponding to 1.5×10^{-13} cm, the correction is 42 percent, which gives $\sigma=10\times10^{-28}$ cm^2. Similar results hold for other types of interaction. For r_0 about 1.0×10^{-13} cm the correction is $(2/3)(0.42)=28$ percent, or $\sigma=9\times10^{-28}$ cm^2.

Considerations based on the WKB method seem to indicate that the effect of any reasonable potential function $v(r)$ will lead to the same essential result, namely, an increase in σ for a finite range of interaction.

These results for σ are just on the verge of the upper limit allowed by Chadwick and Goldhaber, and hence definite conclusions are to be avoided. It is only suggestive that with slightly improved experimental technique, an upper limit may be fixed for r_0.

It seems unlikely that any experiment of this kind can offer definite information about the shape of the interaction in the light of present ideas, since σ is so insensitive to the particular form of interaction chosen.

HARVEY HALL

New York University,
University Heights,
February 12, 1936.

[1] Bethe and Peierls, Proc. Roy. Soc. **A148**, 146 (1935).
[2] Chadwick and Goldhaber, Proc. Roy. Soc. **A151**, 479 (1935).

The Scattering of Protons by Protons

We have examined the scattering of protons by protons at 5° intervals from 15° to 45° and at voltages from 320 to 980 kilovolts, using a linear amplifier connected to an ionization chamber which, with its attached slit system, can be oriented at will with respect to a narrow proton beam in a scattering chamber containing palladium-purified hydrogen at 12.0 mm pressure. From 100 to 2000 scattered protons were observed at each point. The angular definition is about 2°. Voltages are held constant to one percent and measured to two percent by a corona-free high resistance voltmeter (1000 ten-megohm resistors). Range limitations prevent observations at the highest angles for voltages below 600 kilovolts.

Data taken at fixed voltages (630, 696, 740, 830, 922, and 980 kilovolts at scattering volume) with variable angle are consistent and in agreement with independent observations at fixed angles (15, 20, 25, 35, and 40 degrees) and variable voltage. All comparisons are made *on an absolute basis*, the largest error (10 to 20 percent) arising from difficulties connected with the continuous absolute determination of the primary proton current, which cannot be measured with an ordinary Faraday cage due to ionization in the hydrogen gas. Our observations do not confirm White's report[1] that the scattering varies with angle from one through one-quarter to nine times Mott's values for proton-energies of 600 to 750 kilovolts. For this voltage range most of our values lie within 25 percent of those predicted by Mott's formula, although at 740 kilovolts and 45° our value is approximately 50 percent higher than Mott's. As the voltage is increased, consistently larger deviations are in evidence at high angles, our values at 980 kilovolts reaching 5.2 and 7.5 times the Mott values at 40° and 45°, respectively.

M. A. TUVE
N. P. HEYDENBURG
L. R. HAFSTAD

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.,
February 16, 1936.

[1] M. G. White, Phys. Rev. **47**, 573 (1935).

The Ionization Probability of He^{++}

At a meeting of the American Physical Society in 1932 some results were presented on the probability of producing doubly charged helium at a single electron impact.[1] No other publication has been given for this work, but in view of a large number of inquiries concerning these results we thought it worth while to repeat the measurements on a different instrument and present them in this journal.

The shape of the probability function, Fig. 1, was found

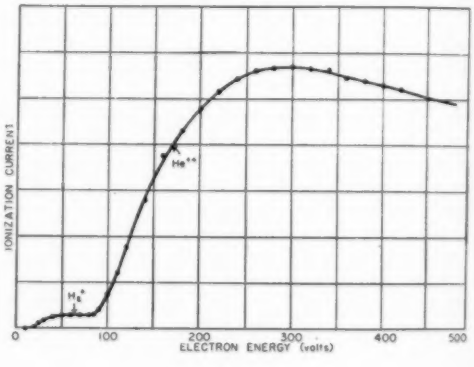

FIG. 1.

by setting the mass spectrograph on the $m/e=2$ peak and varying the electron velocity. A small impurity of molecular hydrogen was present but at about 79 volts an abrupt rise in the curve began which can only be interpreted as He^{++}. At 300 volts the current due to this ion is 1.1 percent of that ascribed to He^+ which means that at this point only 0.55 percent of the ions are doubly charged. The absolute values may be determined by referring to the data of Smith[2] on the singly charged ion.

WALKER BLEAKNEY[3]
LINCOLN G. SMITH

Palmer Physical Laboratory,
Princeton, New Jersey,
February 12, 1936.

[1] W. Bleakney, Phys. Rev. **43**, 378 (1933).
[2] P. T. Smith, Phys. Rev. **36**, 1293 (1930).
[3] We wish to make acknowledgment of financial assistance from the Penrose fund of the American Philosophical Society.

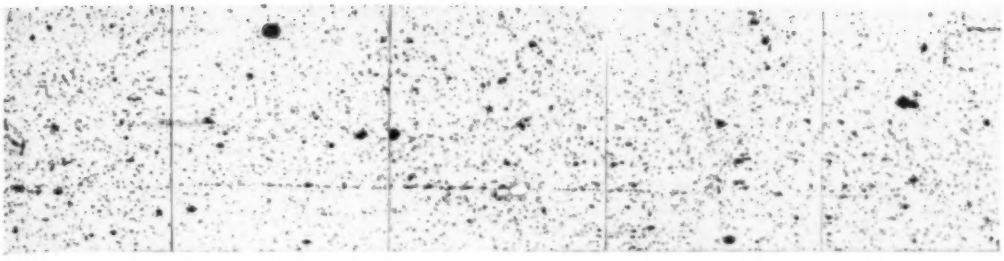

FIG. 1.

Direct Photographic Tracks of Atomic Cosmic-Ray Corpuscles

Within the last year or two there has been general agreement that at least to a very considerable extent the cosmic rays entering the upper regions of our atmosphere are very high speed particles. Anderson and Neddermeyer[1] have recently reported proton tracks secured in a cloud chamber at the top of Pike's Peak and Herzog and Scherrer[2] have similarly secured a track in a cloud chamber on the Jungfrau in the Swiss Alps. The latter workers gave the length of the track as 18 cm and its energy equivalent as 45 MEV.

Fig. 1 shows the track of a particle in a special photographic emulsion sent up on the recent National Geographic-United States Army Stratosphere Flight. The length of the track in the emulsion is approximately 350 grains and the energy absorbed in the emulsion approximately 100 million electron volts. Part-way along the path there is apparent the track of an ejected particle which is presumably a proton. The main track is interpreted as being due to an alpha-particle on the basis of the comparative response of this emulsion to protons and alpha-rays. The stereoscope shows that the main track is well imbedded in the emulsion and that the track of the ejected particle rises quite steeply and comes out at the surface. A simple tool is provided by this technique for the study of cosmic rays at high altitudes.

T. R. WILKINS
H. ST. HELENS

University of Rochester,
Rochester, New York,
February 6, 1936.

[1] Presented at the meeting of the American Physical Society, St. Louis, January 2, 1936.
[2] Herzog and Scherrer, Ann. de physique **4**, 489 (1935).

X-Ray Diffraction by a Film of Counted Molecular Layers

Dr. Irving Langmuir and Dr. Katharine Blodgett have kindly given us one of their films of multiple layers of barium stearate, from which we have been studying x-ray diffraction. Our film, prepared by Dr. Blodgett, has 301 molecular layers deposited on glass. According to Dr. Blodgett,[1] alternate layers of molecules reverse directions, thus bringing the heads of the molecules in adjacent layers together to form a reflecting plane. The grating space is thus the thickness of two molecular layers. She measured this distance by optical means and found it to be approximately 48.8A.

The photographs of Fig. 1 show several of the L and M lines of tungsten obtained by using this artificial crystal in a vacuum spectrograph, arranged for Bragg focusing. The first three orders of the L series are shown on the same plate, different times of exposure being given to the different orders. The relative intensities of the first, second and third orders are found to be 100 : 28 : 31, respectively. The lines are found to be not quite as sharp as those obtained from a gypsum crystal, but in the case of the M line, almost as sharp as theoretically possible from the limited number of molecular layers.

FIG. 1.

It is evident that the use of such a film, with a known number of layers and a known distance between the reflecting planes, may serve as an accurate method of measuring the absolute wave-length of x-rays, and hence of Avogadro's number and the charge on the electron. An experimental study is also made possible of the dependence of widths of x-ray lines upon the number of cooperating layers.

We wish to thank Professor A. H. Compton and Dr. Elmer Dershem for calling these possibilities to our attention.

CLIFFORD HOLLEY
SEYMOUR BERNSTEIN

Ryerson Physical Laboratory,
University of Chicago,
February 15, 1936.

[1] Katharine B. Blodgett, J. Am. Chem. Soc. **57**, 1007 (1935).

Evidence for the Enrichment of Carbon in the Heavier Isotope by Diffusion

An apparatus similar to that described by Hertz[1] for the separation of gaseous isotopes by diffusion has been constructed by one of us (D.E.W.) and used to produce methane gas in which the relative abundance of C^{13} is several times its value in normal methane.

The diffusion apparatus[2] consists of thirty-five "separation members," each involving its own circulating mercury diffusion pump and auxiliary porous tubing. Starting with twenty-five liters of pure methane gas at a pressure of 8 mm of mercury, twelve hours of circulating the gas suffices to establish an isotopic concentration equilibrium in the apparatus, at which time the enriched sample (200 cc at 6 mm pressure) may be removed for analysis. Calculations, based upon mass spectrometer measurements of the concentration factor achieved with the isotopes of neon, indicate that, under favorable conditions, this end-product might be expected to contain as high as fifteen or twenty atomic percent of C^{13}. For the abundance in ordinary carbon Jenkins and Ornstein[3] obtained 0.95 ± 0.1 percent by intensity measurements on the band spectrum, while Aston[4] obtained 0.71 ± 0.07 with the mass spectrograph.

Spectroscopic observation, on the diffused sample were made by the second author (F.A.J.) and gave evidence for a considerable enrichment in C^{13}. About $\frac{1}{2}$ mm of the methane was introduced into a small discharge tube containing 10 mm of purified argon. This gave an intense source of the Swan system of C_2 bands. Fig. 1 shows a comparison of the C_2 bands from ordinary methane (upper) and from the enriched sample (lower). The head marked by a single dot is the 1,0 band, λ4737, of $C^{12}C^{12}$. The corresponding head of $C^{12}C^{13}$ at λ4744, marked by two dots, is considerably more intense in the lower spectrum. Three measurements of the relative intensity of λ4744 to λ4737 were made from different exposures by means of density marks photographed on the same plate, and gave the values 0.143, 0.139 and 0.143. Adopting 0.142, this corresponds to an abundance of 6.6 atomic percent of C^{13}, or an enrichment by a factor of at least 7. That this is considerably less than the calculated value given above is not to be taken as significant, however, since the conditions for the diffusion in which this sample was obtained were not such as to give the most efficient separation. Furthermore, although precautions were taken to exclude ordinary carbon from the discharge tube, a small amount of it would considerably affect the result.

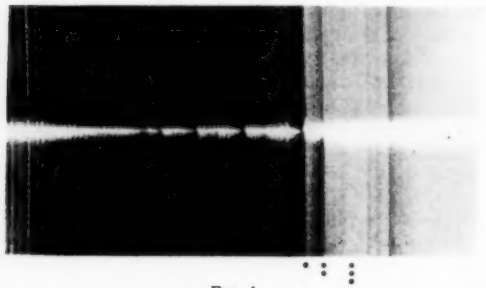

FIG. 1.

An interesting confirmation of this determination is the appearance on the spectrogram of a faint head (marked by three dots) in the calculated position for $C^{13}C^{13}$. In ordinary carbon, this has an intensity less than one ten-thousandth of that of the main band, and is not observable in emission. In stellar absorption spectra, however, it has been identified by Sanford.[5] If the above measurement is correct, its intensity for the enriched sample should be one two-hundredth, and this is enough to make it barely visible. There can be no doubt of its identity here, since it does not appear in the spectrum of ordinary carbon.

We wish to express our thanks to Drs. B. H. Sage and W. N. Lacey, of the Chemistry Department of the California Institute of Technology, for kindly supplying us with the very pure methane gas required for this work. To Dr. Wm. R. Smythe of the Physics Department, at whose instigation the separation of isotopes was undertaken here, is due more than an ordinary acknowledgment for the many valuable suggestions he has made in connection with the construction and operation of the diffusion apparatus. We are also indebted to Dr. H. T. Byck for assistance in the spectroscopic work.

DEAN E. WOOLDRIDGE

Norman Bridge Laboratory of Physics,
California Institute of Technology,
Pasadena, California,

F. A. JENKINS

Department of Physics,
University of California,
Berkeley, California,
February 8, 1936.

[1] G. Hertz, Zeits. f. Physik **79**, 108 (1932); Naturwiss. **21**, 884 (1933).
[2] To be more fully described in a later paper.
[3] F. A. Jenkins and L. S. Ornstein, Proc. Amst. Acad. Sci. **35**, 3 (1932).
[4] F. W. Aston, Proc. Roy. Soc. **A149**, 400 (1935).
[5] R. F. Sanford, Publ. Astronom. Soc. Pacific **41**, 271 (1929).

Two-Body Problem in Gene.al Relativity Theory

In a recent paper,[1] Silberstein attempts to show the incorrectness of the general theory of relativity. His reasoning is as follows:

> a. I set up a static solution of the gravitational equations which has two singular points and is everywhere else free from singularities.
> b. The two particles so represented are not accelerated in each other's gravitational field, in contradiction with experience. Hence the gravitational equations of the general relativity theory are incorrect.

We should like to point out the following. Even if (a) were the case the conclusion (b) would not be justified. For in a field theory only a representation of masses which is free from singularities can be accepted, since at a singularity the laws of the field are violated. However the assertion (a) is not correct. We shall show that the solution given by Silberstein has singularities outside of the two points. This we did not notice in our recent paper, where we referred to this solution, which had been previously communicated to one of us.

In order that a line element of the form (1) (cf. Silberstein's paper) represent a regular gravitational field outside

of the two particles, not only must ν and λ and their first derivatives be continuous, but λ must vanish everywhere for $x_1=0$ except at the two mass-points. The latter condition is necessary in order that an infinitesimal circle in the plane $x_2=$const., $x_4=$const. with center at $x_1=0$ shall have the ratio of circumference to diameter, each measured in natural length (integral of ds), equal to π. One can show that the solution in question does not satisfy these conditions as far as the funcion λ is concerned.

For this purpose we put the expression (10) into a more convenient form by introducing the angle α formed by the two radii-vectors r_1 and r_2. This satisfies the relation

$$r_1 r_2 \sin\alpha = Dx_1.$$

The bracketed expression in (10) can now be written

$$[\ ] = \pm\cos\alpha - 1,$$

where the double sign comes from the square root. According to Silberstein the square root is always to be taken positive, so that one should therefore write

$$[\ ] = |\cos\alpha| - 1.$$

This obviously leads to discontinuous derivatives for λ on the surface $\alpha=\pi/2$, in violation of the conditions for regularity.

As a matter of fact, a closer investigation shows that the calculation can be carried through without the introduction of the square root and the resultant ambiguity in sign. One then finds that in the correct solution

$$[\ ] = \cos\alpha - 1.$$

This, however, also fails to satisfy the regularity conditions, for λ is nonvanishing on the axis ($x_1=0$) between the two mass-points.

We should like to remark that, as shown in a letter to one of us, Professor C. Lanczos of Purdue University has independently recognized the error in Silberstein's paper.

A. EINSTEIN
N. ROSEN

Institute for Advanced Study,
Princeton, New Jersey,
February 17, 1936.

[1] Silberstein, Phys. Rev. **49**, 268 (1936).
[2] Einstein and Rosen, Phys. Rev. **48**, 73 (1935).

A Correction to "On the Structure and Interpretation of the Infrared Absorption of Crystals"[1]

R. B. Barnes, R. R. Brattain and the author would like to express their thanks to Dr. M. A. Durand for calling their attention to a typographical error in their paper of the foregoing title. In Table I on page 590, the columns C_{12} and C_{44} were interchanged. The discussion in the text is not to be altered.

FREDERICK SEITZ

University of Rochester,
Rochester, New York,
February 10, 1936.

[1] Barnes, Brattain and Seitz, Phys. Rev. **48**, 582 (1935).

Infrared Absorption of Hydrogen Chloride in Nonionizing Solvents

The appearance of the interesting paper of E. K. Plyler and D. Williams on the infrared absorption spectrum of hydrogen chloride in benzene at about 3.4μ[1] has suggested to us the propriety of recounting our observations on the infrared absorption of that substance in various solvents, made in continuance of work on the Raman spectra of these solutions.[2] The work has been performed on the harmonic at about 1.76μ by means of a prism-grating spectrometer employing an echelette grating of 3600 lines to the inch. The results are in general agreement with those obtained from Raman spectra—the absorption band is displaced to lower frequencies by an amount increasing with the dipole moment of the solvent, the displacement reaching a limiting value beyond which further increase in moment is without effect. As is perhaps to be expected, there is no trace of rotational structure, though solutions in PCl_3, $POCl_3$, CCl_4, SO_2, show either very distinctly, or at least clear signs of a division into two bands with maxima separated by an amount, in PCl_3 and CCl_4, about the same as the P–R doublet separation in the incompletely resolved gaseous absorption, while in $POCl_3$ and in SO_2 the separation is distinctly greater than in the gas (about 220 cm^{-1} in SO_2, solution, against 120 cm^{-1} for the gas). Moreover the intensity of the low frequency branch is much greater than that of the other, in contradistinction to the intensity relations in the P and R branches in the gas. Diminishing the temperature in PCl_3 solutions makes the division into two bands less pronounced and also causes a relatively small diminution in the frequency of maximum absorption.

The absorption coefficient in these solutions, judged from a comparison of cell lengths required for about the same absorption in the gas and in solution, seems to be only slightly greater than in the gas.

W. WEST
R. T. EDWARDS

Chemistry Department,
Washington Square College, New York University,
February 8, 1936.

[1] Plyler and Williams, Phys. Rev. **49**, 215 (1936).
[2] West and Arthur, J. Chem. Phys. **2**, 215 (1934).

Ultraviolet Absorption in Hydrogen Fluoride. A Correction

An ultraviolet absorption spectrum in hydrogen fluoride was recently reported in a letter by K. Siga and H. Plumley.[1] It has since been demonstrated by the second author that the observed spectrum must be attributed to the presence in the hydrogen fluoride of a trace of sulfur dioxide as an impurity. It is concluded that hydrogen fluoride at atmospheric pressure, and with a path length of 30 cm, shows no absorption in the ultraviolet region above 2000A.

H. J. PLUMLEY

Ryerson Physical Laboratory,
University of Chicago,
February 4, 1936.

[1] K. Siga and H. Plumley, Phys. Rev. **48**, 105 (1935).

On the Slowing Down of Neutrons

". . . It is easily shown that an impact of a neutron against a proton reduces, on the average, the neutron energy by a factor $1/e$."

A proof of the above statement by Fermi has been published recently by Wick.[1] In his derivation Wick considers the logarithm of the energy instead of the energy itself. The reason for this choice, according to Wick, is that for the logarithm the most probable value is approximately equal to the average value. The average of the logarithm of the energy is, however, not equal to the logarithm of the average energy and thus the original statement cannot be considered as strictly correct.

As an illustration let us first consider a single collision. Geometrical considerations show that the probability that the neutron after collision flies away under an angle between θ and $\theta+d\theta$ with its original direction is given by

$$P(\theta)d\theta = 2\sin\theta\cos\theta d\theta.$$

Denoting the most probable value with an asterisk and the average value with a bar we find

$$\theta^* = 45° \qquad \bar{\theta} = 45°.$$

The energy E_1 retained by the neutron, expressed in terms of the original energy E_0, is equal to

$$E_1 = E_0\cos^2\theta.$$

For the energy distribution follows: $P(E_1)dE_1 = dE_1/E_0$, E_1^* all values equally probable, $\bar{E}_1 = E_0/2$. For the logarithm of the energy, introducing $x_1 = \log(E_0/E_1)$, we find

$$P(x_1)dx_1 = e^{-x_1}dx_1, \quad x_1^* = \log(E_0/E_1)^* = 0, \quad \bar{\bar{x}}_1 = 1.$$

The velocity distribution is given by

$$P(v_1)dv_1 = (2v_1/v_0^2)dv_1, \quad v_1^* = v_0, \quad \bar{v}_1 = \tfrac{2}{3}v_0.$$

After n collisions the distribution probabilities are

$$P(E_n)dE_n = \frac{\{\log(E_0/E_n)\}^{n-1}}{(n-1)!}\frac{dE_n}{E_0}, \quad E_n^* = 0, \quad \bar{E}_n = E_0/2^n.$$

$$P(v_n)dv_n = \frac{2^n\{\log(v_0/v_n)\}^{n-1}}{(n-1)!}\frac{v_n dv_n}{v_0^2},$$

$$v_n^* = v_0/e^{n-1}, \quad \bar{v}_n = v_0/(3/2)^n.$$

$$P(x_n)dx_n = \{e^{-x}nx_n^{n-1}/(n-1)!\}dx_n, \quad x_n^* = n-1, \quad \bar{x}_n = n.$$

The last line represents Wick's result.

The above formulas show definitely that the average value of the energy after n collisions is $1/2^n$ of the original energy. It is clear that the difference between $1/2^n$ and $1/e^n$ is of little importance in most applications. Moreover, it is of course possible to justify the use of $1/e^n$ by wording the original statement differently. It may be shown, namely, that after n collisions about half of the neutrons will have energies less than E_0/e^n.

S. Goudsmit

Department of Physics,
University of Michigan,
January 31, 1936.

[1] Wick, Phys. Rev. **49**, 192 (1936).
Added to proof: A detailed derivation of the energy distribution for collisions with protons for heavier nuclei has just been published by E. U. Condon and G. Breit, Phys. Rev. **49**, 229 (1936).

The Theory of Fusion

Herzfeld and Mayer have recently given in this journal[1] a straightforward theory of the fusion process in solid argon and helium in which they take as the determining criteria for the instability leading to melting, the equations

$$(\partial^n F/\partial V^n)_T = 0 \qquad n = 1,2$$

where F is the Helmholtz free energy of the solid. In studying the problem directly from the point of view of the partition function the following points have suggested themselves:

For a Debye oscillator having $\nu < kT/h$, the partition function increases rapidly with T, being of the form $[\exp(-h\nu/2kT)]/[1-\exp(-h\nu/kT)] \cong kT/h\nu$, an effect which is further enhanced by the lowering of the oscillator frequencies with increase of volume, and possibly also by the existence of maxima in the density distribution of oscillators.[2] As the temperature is raised these low frequency oscillators begin to contribute heavily to the free energy, although not so much to the total energy. One can then argue that there is an instability leading to the dissociation of some of the low frequency oscillators, corresponding to some type of breaking up of the solid into smaller units, which in themselves maintain much the same molecular grouping and structure as in the solid. This is a point of view which has been much discussed of late in view of the x-ray studies of Stewart and others. While a detailed theory is not yet available, it appears that the average diameter of such a small "elementary" region should be of the order of magnitude of nhc/kT_m where c is the velocity of sound in the solid, T_m the melting temperature, and n some small integer whose function is to determine the order or type of oscillator which becomes unstable. The actual theory is complicated by the related effects of the anharmonicity and possible resonance between different oscillators.

Various attempts have been made to estimate the relative volume of the liquid state which may be considered to be still roughly in the same condition as the solid, but no really reliable results have been found. Values of 50 percent and above do not seem impossible. This would not mean of course that such great regularity would show up in x-ray diffraction measurements due to reorientations of the small elements and secondary distortions within them.

While a theory of the above type cannot be expected to yield a definitive interpretation of instability in crystals, yet within the limits of a calculation comparable to that of Herzfeld and Mayer it seems to offer some definite promise for an extension of their theory to include the concept of elementary regions of high molecular regularity in the liquid state.

E. L. Hill

Department of Physics,
University of Minnesota,
February 13, 1936.

[1] Herzfeld and Mayer, Phys. Rev. **46**, 995 (1934).
[2] Cf. M. Blackman, Proc. Roy. Soc. **A148**, 365 (1935) and later papers in the same journal.

Proceedings of the American Physical Society

Minutes of the St. Louis Meeting, December 31, 1935–January 2, 1936

THE 37th Annual Meeting (the 203rd regular meeting) of the American Physical Society was held at Washington University, St. Louis, Missouri, on Tuesday, Wednesday, and Thursday, December 31, 1935 and January 1 and 2, 1936 in affiliation with Section B—Physics—of the American Association for the Advancement of Science and the American Association of Physics Teachers. The presiding officers were Professor R. W. Wood, President of the Society, Dean F. K. Richtmyer, Vice President, and Professor L. A. DuBridge, Dr. W. E. Forsythe, Professor A. L. Hughes, Professor L. R. Ingersoll, Professor D. C. Miller, Professor H. M. Reese, Professor Alpheus W. Smith, and Professor D. L. Webster. There were about three hundred physicists in attendance at the meeting. All sessions were held in the Wayman Crow Hall of Physics of Washington University.

The joint session with the American Association of Physics Teachers was held on Tuesday afternoon at two o'clock in Crow Hall in a symposium on *Photoelectricity* at which Professor D. L. Webster, President of the American Association of Physics Teachers presided. The invited papers were as follows: (1) *The New Physics and the Undergraduate* by Professor A. A. Knowlton, Reed College; (2) *Photoconductivity in Crystals* by Professor A. L. Hughes of Washington University; and (3) *The Copper-Copper Oxide Barrier Layer* by Dr. L. O. Grondahl of the Union Switch & Signal Company.

On Tuesday evening the Physical Society and the American Association of Physics Teachers held a dinner at the Gatesworth Hotel. Professor R. W. Wood, President of the Physical Society, presided. Brief speeches were made by President Wood, President D. L. Webster of the Association of Physics Teachers, and Professor A. L. Hughes, Chairman of the Local Committee for the Physical Society, the Association of Physics Teachers, and Section B of the A.A.A.S. Dr. Thomas C. Poulter, second in command of the recent Byrd Antarctic Expedition, spoke on *The Byrd Expedition in the Field*, with special reference to its scientific work, and showed motion pictures of the life and work of the expedition. There were two hundred and twelve present at the dinner.

Annual Business Meeting. The regular annual business meeting of the American Physical Society was held on Wednesday morning, January 1, 1936, at nine-thirty o'clock, President Wood presiding. The President had appointed Messrs. K. K. Darrow and F. S. Goucher tellers to canvass the ballots for the officers of the Society. In accordance with the report of the tellers the following elections were declared made for the year 1936:

President	F. K. Richtmyer
Vice President	H. M. Randall
Secretary	W. L. Severinghaus
Treasurer	G. B. Pegram
Members of the Council—four-year term	J. W. Beams
	E. C. Crittenden
Members of the Board of Editors—three-year term	H. A. Bethe
	L. A. DuBridge
	M. A. Tuve

The Secretary reported that during the year there have been 148 elections to membership. The deaths of 13 members have been reported; 19 have resigned, and 54 have been dropped. The membership of the Society as of December 29, 1935 was as follows: 2146 members; 694 fellows; 6 honorary members; total 2846.

The Treasurer presented a summary of the financial condition of the Society. It was impossible to present a final report for the year at the Annual Business Meeting because the fiscal year ends on December 31. The Treasurer's financial report will be audited, printed, and sent to members.

The Managing Editor reported that a preliminary estimate of the cost of publishing the *Physical Review*, *Physics*, and the *Reviews of Modern Physics* for 1935 would probably be under $5.00 per member; that the relatively low

cost for this year was due to the following: (1) The subscription income was greater than anticipated partly because of the putting into effect of the policy of charging for the additional postage for foreign subscribers and partly because the number of subscribers showed a healthy increase, and (2) the volume of publication remained practically the same as in 1934. It was emphasized that this fortunate state of affairs might not continue for long and that it would require only a relatively small increase in the number of pages to be published to run the cost above the resources of the Society.

The joint session with Section B of the A.A.A.S. and the American Association of Physics Teachers was held on Wednesday morning at ten o'clock. The presiding officer was Professor John T. Tate, Vice President of Section B. The Retiring Vice President of Section B, Dean Henry G. Gale, presented a paper on *The Diffraction Grating*. Professor R. W. Wood, President of the Physical Society delivered an address on *Optical and Physical Effects of High Explosives* and Professor Arthur H. Compton spoke on *Recent Developments in Cosmic Rays*.

On Thursday afternoon at four-thirty o'clock the Physical Society joined with the American Mathematical Society in the Municipal Auditorium at the Josiah Willard Gibbs Lecture which was delivered by Professor Vannevar Bush of the Massachusetts Institute of Technology on *Mechanical Analysis*. On the same afternoon Dr. V. K. Zworykin of the Radio Corporation of America gave a lecture on *Electron Optical Systems and Their Applications*.

Meeting of the Council. At the meeting of the Council held on Wednesday afternoon, January 1, 1936 one candidate was elected to fellowship and ten candidates were transferred from membership to fellowship. Forty candidates were elected to membership. *Elected to fellowship:* Edward Teller. *Transferred from membership to fellowship:* Norman I. Adams Jr., A. B. Cardwell, Floyd A. Firestone, George M. Murphy, Lothar Nordheim, Leland B. Snoddy, E. C. Stevenson, James D. Stranathan, J. C. Street, and E. J. Workman. *Elected to membership:* Lewis G. Abernathy, C. J. Bakker, Alfred Boch, William M. Breazeale, John H. Buck, Glen D. Camp, C. Fred Clarke, A. W. Dicus, Charles O. Duevel, Jr., Douglas H. Ewing, G. M. Giannini, Walter Gordy, Armin Helz, Robert A. Howard, Harry R. James, Martin Katzin, William G. Keck, Richard C. Keen, Chris. P. Keim, Harry C. Kelly, Philip G. Koontz, Elbert P. Little, Robert J. Maurer, Frederic D. Merrill, Jr., Robert D. Miller, Yukihiko Miyagawa, Keron C. Morrical, Wilson C. Morris, Richard G. Nuckolls, Kinnosuke Ogura, Hidesuke Ohmata, Anna L. Peckham, James A. Peoples, Jr., Paschal H. Pretz, P. Scherrer, Edward J. Schremp, Francis W. Sears, J. W. Smith, Harris M. Sullivan, and D. H. Tomboulian.

The regular scientific program of the Society consisted of seventy-five contributed papers. Numbers 7, 10, 17, 36, 39, 42, 43, 64, and 65 were read by title. The abstracts of the contributed papers are given in the following pages. An *Author Index* will be found at the end.

G. B. PEGRAM, *Secretary pro tempore*

ABSTRACTS

1. Dia- and Paramagnetic Susceptibilities at Two to Ten Oersteds. WILLIAM SCHRIEVER AND R. E. SMITH, *University of Oklahoma.*—Specimens of three pure diamagnetic elements and three pure paramagnetic salts were investigated by measuring the force on a spherical specimen when it was placed in an axially symmetrical field whose strength, and space rate of change of strength along the axis of symmetry, could be calculated. The forces varied from 6 to 25×10^{-6} dyne and the sensitivity of the apparatus was 10^{-6} dyne per millimeter deflection at a scale distance of 317 cm. The small deflections were reproducible to within 6 percent, the large to within 1 percent. The necessary variation of the field over the specimen was less than 5 percent of the mean value of the field strength. The observed variations are greater than the experimental errors. At one oersted $FeCl_3$ had a susceptibility of 159.

Substance	Volume of Substance (cm^3)	Volume Susceptibility (e.m.u.)			
		$H=2.5$	$H=5$	$H=10$	Int. Crit. Tab.
Bismuth	1.32	−10.9	−11.2	−10.7	−13.78
Antimony	1.55	−4.45	−3.89	−3.10	−5.75
Cadmium	1.59	——	−0.895	−0.63	−1.56
Bismuth (Powder)	0.992	−9.78	−7.65	−7.13	——
(Powders)		$H=2$	$H=3$	$H=5$	
$FeCl_3$	0.43	153.	148.	140.	79.
$FeSO_4+7H_2O$	0.505	78.5	77.7	75.	72.5
$Co_2(SO_4)_3+7H_2O$	0.435	69.5	79.8	75.	240.

2. Magnetic Viscosity. C. W. HEAPS, *The Rice Institute.*—Barkhausen jumps of magnetization in nickel and iron may continue to occur for 20 or 30 seconds after the magnetizing force has become constant. This phenomenon appears to be a type of magnetic viscosity which cannot be caused directly by eddy currents. A possible explanation may lie in highly localized regions of high temperature, these high temperatures being produced by the localized eddy currents associated with single Barkhausen reversals. The flow of heat from these small regions would cause a redistribution of local strains and thus allow other regions which are not very stable magnetically to make a Barkhausen jump. A wave of magnetic reversal may be assumed to penetrate an elementary domain of volume 10^{-6} cm^3 according to the same laws observed by Sixtus and Tonks for very large Barkhausen discontinuities. It appears that on this assumption sufficiently high local temperatures will not be produced to explain the phenomenon in the manner indicated. However, by making the reversal time of the elementary domain extremely small a local high temperature of sufficient magnitude might be produced. A second factor to be considered is the uneven distribution of the large scale eddy currents produced by the change of magnetic force. These large scale currents will warm the outside layers of the wire more than the inner layers and thus some time will be required for the attainment of a uniform temperature. There are reasons for believing that the large scale eddy currents do not by their heating effect have much influence on magnetic viscosity.

3. A Prerecording Oscillograph and Its Use in Studying Rectifier Operation. A. W. HULL AND HANS LAUB, *General Electric Company.*—By utilizing the phosphorescence of the screen of a cathode-ray oscillograph it is possible to record electrical conditions just prior to, as well as during and after, an unpredictable event, such as arc-back of a rectifier. The oscillograph is allowed to write continuously on its screen, and the image fades as fast as it is written, except for a slight time lag due to the decay period of the phosphorescence. This time lag constitutes a memory of what happened just prior to any given instant. A camera whose shutter is operated, with the help of a Thyratron, by the event being studied, sees and records this "memory" as well as the trace that is written while the shutter is open. We have called these records *memory oscillograms.* With the help of these oscillograms we have obtained, for the first time, reliable information on the question: "At what part of the cycle does a rectifier arc-back?" With Phanotrons and Thyratrons, all the arc-backs that we have observed occur at the middle of the inverse cycle, i.e., when the voltage is maximum.

4. The "Effective" Capacity of an Intermittent Glow Discharge Tube. DANIEL E. CLARK AND LESTER I. BOCKSTAHLER, *Northwestern University.*—When the electrodes of a discharge tube are joined in parallel with a capacity and supplied with a constant current, a regular periodic discharge takes place. The phenomena may be explained in terms of the charging and discharging of a condenser. The magnitude of the capacity which must be assigned to the condenser in order to calculate the period of the discharge is greater than the value of the external capacity plus the circuit capacity computed from its dimensions. The additional "effective" capacity was found for tubes with half-inch aluminum electrodes, 2 cm separation, by using, in turn, H_2, He, N_2, O_2, A and air as residual gas. The magnitude of the "effective" capacity ranged from 120 micromicrofarads for H_2 to 600 for N_2. For H_2, O_2, A and air the "effective" capacity was independent of the current while for He and N_2 there was a slight increase with current. The period is a linear function of the external capacity. No simple relation exists between the gas pressure and frequency save that the latter increases as the former decreases. The frequency was found by means of a Duddell oscillograph which recorded the amplified output of a photoelectric cell which responded to the light flashes in the discharge.

5. Sensitivity of Photon Counters. S. W. BARNES AND L. A. DUBRIDGE, *University of Rochester.*—It is well known that Geiger-Müller tube counters with cylinders of Ni, Mo, Au, etc., are sensitive to ultraviolet light, counts being produced by the release of electrons from the metal surface. There have been few satisfactory measurements of the sensitivity, however, and almost no attempt to realize the greatest possible sensitivity. It is probable that no counters have been used capable of giving a measurable response to ultraviolet light intensities less than about 3000 quanta/cm^2 sec. Claims for higher sensitivity appear to be open to question. In the study of the supposed mitogenetic rays there is a demand for counters responding to 2–5 quanta/cm^2 sec. The sensitivity limit is set (1) by the presence of background counts and (2) by the photoelectric yield of the surface. Because of (1) it will be difficult to measure radiation which yields less than 2 electrons/min. The photoelectric yield for most metals at 2536A is of the order of 10^{-5} electron/quantum or less, giving the sensitivity limit of 3000. However, a distilled Mg surface in vacuum may have a yield of 10^{-2} which should give a counter sensitivity of 3 quanta/cm^2 sec. Unfortunately most counting gases destroy the active surface. A mixture of pure He and Ne does not affect the surface but gives rather poor counts. A counter of this type will be described.

6. The Geiger-Müller Photon Counter—Quantitative Aspects, Detection of Small Intensities, Use in Absorption Spectroscopy. WALTER D. CLAUS AND ALEXANDER HOLLAENDER, *University of Wisconsin.*—The authors have recently discussed[1] some of the uncertainties involved in the calibration of the Geiger-Müller photon counter. An approximate solution to the problem is obtained through the intermediary of the counter tube used as a calibrated vacuum photo-cell. The photoelectric yield of the sensitive surface of the counter is definitely less than the yield of the same surface (Zn, Cu, Al, sensitized, not outgassed) in vacuum, so that $10^2 h\nu$/cm^2/sec. is the lower limit of intensity in the ultraviolet (2300A) which can be detected with the counters thus far investigated. Attempts to detect mitogenetic radiation from a number of sources

with these counters have been made. The quantitative response of the photon counter to incident radiation suggests the use of the device in absorption spectroscopy, while its response to extremely small intensities permits the use of very pure light from readily available, constant sources of continuous radiation. The instrument should thus be of great value in the study of fine structure in absorption spectra.

[1] Claus and Hollaender, J. Opt. Soc. Am. **25**, 270 (1935).

7. Fowler's Photoelectric Theory, Assuming Quantum Absorption Probability a Function of Electronic Energy. ALAN T. WATERMAN, *Yale University*.—It is assumed in Fowler's theory that the photoelectric current is proportional to the number of "available" electrons, i.e., those with energies normal to the surface which exceed a given value. This implies that the chance of an electron absorbing a quantum is independent of its energy. Recent theoretical progress following quantum-mechanical lines does not justify this assumption but indicates that this chance depends upon the electron's momentum normal to the surface. Therefore this feature is incorporated into Fowler's treatment by including a factor proportional to the normal kinetic energy in the expression for the number of "available" electrons. On integrating it is found that the form of the Fowler function, $F(x)$, used in evaluating the work function, remains unaltered and leads to the same value of the work function, computed by horizontal displacement of the experimental curve. The vertical shift alone is affected. The same result is obtained on assuming a quantum absorption probability proportional to the first power of the normal velocity instead of the second. One may conclude therefore (1) that there is no inconsistency between the Fowler analysis (as used hitherto) and recent quantum-mechanical theory in the respect mentioned, and (2) that apparently the Fowler theory can give no evidence as to the probability of absorption of a quantum unless vertical shifts are analyzed. The form of the Fowler function $F(x)$ is determined only by the form of the statistical partition function.

8. On the Nature of the Barrier Layer in the Cuprous Oxide Photovoltaic Cell. II. J. W. BALLARD AND E. HUTCHISSON, *University of Pittsburgh*.—In the course of extended study of cuprous-oxide photo-cells many data were obtained which bear on the behavior of such cells. While some of these results are merely confirmation of the work of previous investigators, many of them are new and significant in the explanation of the action of barrier-layer cells. The following conclusions were drawn from the experimental results: (1) Sensitization of cuprous-oxide surface occurs on bombarding the oxide surface with positive ions, negative ions, or electrons. (2) Sensitization has not been obtained by heating the cuprous-oxide disk as a whole. (3) The positions of maximum spectral responses and of limiting wave-length in the near infrared are independent of the kind of gas used in bombardment. (4) Electrical resistance and total response of a cell depend upon the kind of gas used in bombardment. (5) The maximum sensitization depends upon an optimum time of bombardment. (6) The electrical resistance seems to increase with time of bombardment. (7) The position of maximum spectral response is independent of the time of bombardment. (8) The position of maximum spectral response is probably independent of the kind of metallic film. (9) Spectral transmission of the oxide seems related to the spectral response maxima in the near infrared.

9. Photoelectric Properties of Sodium Films on Aluminum. JAMES J. BRADY AND VINCENT P. JACOBSMEYER, S.J., *Saint Louis University*.—Previous work[1] has shown that thin films of potassium, rubidium, and caesium deposited on a silver surface give a maximum sensitivity when the calculated film thickness is in the order of 10 molecular layers or less. For thicker films, the sensitivity is less. Sodium films are deposited on aluminum, and it is found that the maximum sensitivity is not attained until the film thickness is in the order of 100 molecular layers. The sensitivity is constant for greater thicknesses. No detectable photo-current is observed until the film thickness is in the order of 6 molecular layers. A difference of this kind is expected on the basis of the adion field theory for the work function of the underlying metal is less than the ionization potential of the alkali atom. The reverse is true in the previous work. The photoelectric cell is of the concentric sphere type like that previously reported by Brady.[2] Current-voltage curves are observed for different film thicknesses. These indicate that the work function remains the same for all the film thicknesses investigated. A spectral distribution curve is obtained for the thick film (100 molecular layers). Surface changes after deposition of the sodium are discussed.

[1] Brady, Phys. Rev. **41**, 613 (1932).
[2] Brady, Phys. Rev. **46**, 768 (1934).

10. A Hydrodynamical Theory of Piston Ring Lubrication.* R. A. CASTLEMAN, JR., *East Falls Church, Virginia*.—While a satisfactory hydrodynamical theory of journal bearing lubrication was advanced some 50 years ago by Osborne Reynolds, and has since been considerably extended and applied with gratifying results by Sommerfeld, Kingsbury, Michell and others, much haziness still persists as to the lubrication of the so-called "packing rings" which reciprocate with the pistons of engines. If we assume that the cross section of the ring, no matter how "straight" initially, is soon worn down to some gently rounded form, with the thick part in the middle (e.g., circular, *with very large radius of curvature*) and further assume that oils as used in engine lubrication cannot stand absolute tension, we find that, while the piston is moving at ordinary speeds, the ring will be separated from the wall by a film of lubricant a few ten-thousandths of a centimeter thick, and therefore large compared to the molecular size, so that boundary conditions need not be considered. It also follows that a "small rounding" of the edge of the ring, so often assumed, will not result in the maintenance of a sufficiently thick film. Results from this theory are in qualitative agreement with observations of ring wear by C. G. Williams, and with measurements of ring clearance by F. Salzmann.

* To be read by title.

11. An Analytical Treatment of the Horn-Diaphragm Coupling Chamber for Receiver Measurements. C. K. STEDMAN, *Purdue University.* (*Introduced by K. Lark-Horovitz.*)—A telephone receiver may be tested by removing its horn and loading it acoustically with an air column of variable length in a closed tube.[1] In this way one can measure the acoustic impedance of the horn and the sound power output of the receiver, under the assumption that the air chamber in front of the diaphragm can be regarded as an addition to the tube of an equivalent length r determined by experiment. Under this assumption the acoustic impedance Z_A at the diaphragm is given by

$$Z_A = -j\left(\frac{A_1}{A_2}\right)^2 A_2\rho c \cot kx \qquad (1)$$

where A_1 is the diaphragm area, A_2 the cross section of the tube, and x the total length, i.e., actual length plus r. A complete analytical treatment leads to the formula

$$Z_A = -j\cos^2 kr\left[\left(\frac{A_1}{A_2}\right)^2 A_2\rho c \cot kx\right] - j\left(\frac{A_1}{A_2}\right)^2 A_2\rho c \sin kr \cos kr. \qquad (2)$$

Eq. (1) leads to correct results for the measurement of acoustic impedance of the horn and sound power output, but it leads to quite erroneous results when extended to determine the acoustic impedance at the diaphragm and the mechanical impedance of the diaphragm itself. Eq. (2) covers all cases. Furthermore it is expressed in terms of a single parameter r which can be calculated theoretically at low frequencies, or determined empirically at higher frequencies. The theory has been verified experimentally.

[1] Fay and Hall, J. Acous. Soc. Am. **5**, 46 (1933).

12. Sound Propagation. D. A. BOURGIN, *University of Illinois.*—For a wide class of mixtures marked simplification is possible in the author's earlier formulae for sound propagation in gases. Double transitions and multiple collision effects have been included as required by recent experimental findings. The Knudsen-Kneser expression for gas contamination absorption is shown to be a very special case under the general result, and the implied approximations are brought into evidence. For pure gases the Kneser-Ruttger's method as generalized by Richards is shown to be equivalent in principle with the writer's developments (though the ωi dependence is implicit). C_∞ or $3k/2$ may be introduced by either method.

13. The Direct Detection and Measurement of the Angular Momentum of Light. RICHARD A. BETH, *Worcester Polytechnic Institute and Palmer Physical Laboratory.*—As was first pointed out by A. Sadowsky,[1] Maxwell's theory predicts that a beam of polarized light must, in general, exert a torque on a doubly refracting medium which changes its state of polarization. The predicted value of the effect is quantitatively the same as that obtained by assigning an angular momentum of $\pm h/2\pi$ to left and right circularly polarized photons, respectively, and assuming the conservation of angular momentum. During the past year it was possible to detect and measure this effect.[2] The experiment consists essentially in determining, by mirror and scale, the deflection of a quartz half-wave plate hung from a quartz fiber between two fixed quartz quarter wave plates, the *top* side of the top one being coated with a reflecting layer of aluminum. All plates are horizontal, and the axes of the two fixed plates are 90° in azimuth from those of the hanging plate between them. Light from a tungsten ribbon filament passes through a large Nicol and up into the plate system. The *change* in torque on the half-wave plate was measured when the lower plate was turned back and forth through a fixed angle in resonance and in antiresonance with the natural period of the torsional pendulum. Experimental curves show that the effect varies as required by the theory for different filament temperatures, for different azimuths of the plane of polarization of the Nicol, and for different values of the angle through which the lower plate was turned. The torques obtained were in the region 0.5 to 4×10^{-9} dyne-cm.

[1] A. Sadowsky, Acta et Commentationes Imp. Universitatis Jurievensis **7**, No. 1–3 (1899); **8**, No. 1–2 (1900).

[2] I wish to thank the National Research Council for a grant-in-aid and Professor A. W. Duff, of the Worcester Polytechnic Institute, who originally brought the problem to my attention.

14. Mean Free Paths for Gas Beams in Mercury Vapor. JOHN A. ELDRIDGE, *State University of Iowa.*—The decrease in intensity of a molecular beam as it passes through mercury vapor has been observed and the mean free path (for beams) determined for several gases. The values (in centimeters) at one bar mercury vapor pressure are as follows: He 12.5; H_2 6.85; N_2 5.8; CO_2 5.3; C_2H_6 5.2. These values are slightly smaller than would be predicted from viscosity data on classical lines (using a rigid sphere model). A difference in this direction follows from the wave mechanics.

15. An Attempt to a Purely Theoretical Derivation of the Mass of the Universe. ARTHUR HAAS, *University of Vienna* (now Visiting Professor at Bowdoin College). (*Introduced by W. L. Severinghaus.*)—If a universe of finite mass M consists of N protons+neutrons and R is the radius of the approximately spherical volume, over which the astronomical objects are distributed, we arrive at the relation

$$MRc = 2N^{1}h \quad \text{or} \quad N^{1} = Rm_H c/2h \qquad (1)$$

(m_H the mass of the H-atom).[1] If, on the other hand, we make the assumption that the gravitational energy of the universe is equal, with opposite sign, to the sum of the proper energies of all the material particles contained in the universe, we arrive at the formula

$$N = 5Rc^2/3fm_H, \qquad (2)$$

f denoting the gravitational constant. Combination of (1) and (2) yields

$$N^{1} = (10/3)(hc/m_H{}^2 f) \qquad (3)$$

and hence

$$N = 1.2\times10^{79}, \quad M = 2.0\times10^{55}\text{ g}, \quad (R = 9.3\times10^{28}\text{ cm}) \qquad (4)$$

in agreement with the values derived by Eddington in an entirely different way. No reference, however, to the

ideas of general relativity, of an expansion of the universe, or of a curvature of space and no astronomical data are necessary for the derivation and evaluation of (3). From a comparison with Eddington's theory the relation seems to follow:

$$(e^2/c)\gamma = 2(6/5)^{\frac{1}{2}}h \qquad (5)$$

γ denoting the mass ratio between H-atom and electron. Calculation of e from (5) gives a value which only differs by about one percent from the accepted value.

[1] Cf. A. Haas, Anz. Akad. Wiss. Vienna No. 11, 1932; and another theoretical deduction by K. Sitte and W. Glaser, Zeits. f. Physik **88**, 103 (1934).

16. On the Probability of Radiative Processes for Very High Energies. L. W. NORDHEIM, *Purdue University.*—The present theory of radiative processes (radiation of electrons and pair production by photons) ceases to be valid for fields above the critical limit: potential difference mc^2/e over h/mc. The available formulae (Bethe-Heitler) give accordingly wrong results if applied to cosmic rays. By a method of v. Weizsäcker and Williams the total cross sections can be obtained by summing the contributions of passages at various distances from the atomic nuclei. If the regions are left out, for which the applicability of the ordinary theory is doubtful according to the above criterion, new formulae are obtained which seem to satisfy the observations. They give quantitatively the specific energy losses of electrons in lead (as measured by Neddermeyer and Anderson). In agreement with Compton's analysis only electrons with an initial energy of above 8000 MV have a fair chance to traverse the atmosphere. As at a critical energy (decreasing with the nuclear charge) the cross sections for radiative processes approach zero, the transition effect for showers (greater intensity in air than in lead of the photon radiation accompanying primary electrons) is readily explained, the values of the various absorption coefficients being again at least in qualitative agreement with the experimental data.

17. Mathematical Expression of Charge Distribution in a Space Lattice. VIVIAN JOHNSON, *Purdue University.* (*Introduced by K. Lark-Horovitz.*)—The assumption that the forces holding a solid body together are of an electrical nature has been used with great success by Born, Ewald, and others in the theory of the dynamics of crystal lattices. However, the expression for the potential functions used has never been proved to be convergent. Following a suggestion of W. E. O. Maier, the problem has been treated in a way similar to Riemann's treatment of the Green's function of a rectangular parallelopiped.[1] Thus we prove the convergence of the sum:

$$S = \sum_{-\infty}^{\infty} k \sum_{-\infty}^{\infty} m \sum_{-\infty}^{\infty} n \frac{e^{\pi i q(k+m+n)}}{(k^2+m^2+n^2)^{\frac{1}{2}}},$$

where $q = l/2^p$; p being any integer and l any odd integer. l and p may be chosen so as to approximate any real number. By squaring the denominator of each term the convergence is improved, and therefore also the sum:

$$S' = \sum_{-\infty}^{\infty} k \sum_{-\infty}^{\infty} m \sum_{-\infty}^{\infty} n \frac{e^{\pi i q(k+m+n)}}{k^2+m^2+n^2}$$

is convergent. The electrostatic potential of a lattice distribution can be written as the sum of eight terms, each of the type S', and therefore also is proved to be convergent.

[1] Courant and Hilbert, *Meth. der Math. Phys. I*, pp. 327–330.

18. Energy Distribution of Slow Neutrons. E. U. CONDON AND G. BREIT, *Princeton University.*—The problem of finding the distribution in energy of particles of mass m, initially of the same energy, which have made n impacts with particles of mass M all initially at rest, is solved. It is supposed the impacts are elastic and the distribution in angle isotropic in a coordinate system in which the center of mass is at rest. If x is the ratio of the energy after n impacts to the initial energy then the chance that x lie in dx at x is $(\log 1/x)^{n-1}/(n-1)!$ for $m = M$. For unequal masses the expression is more complicated but easy to calculate. The results have some interest in connection with the slowing of neutrons by elastic impacts with other nuclei, especially with hydrogen nuclei.

19. Does the Alpha-Particle Possess Excited States? EUGENE FEENBERG, *University of Wisconsin.*—Two Hamiltonian operators have been investigated in an attempt to answer the above question. In the first only neutron-proton interactions are considered. These are represented by a potential function of the ordinary (Wigner) type with the explicit form $Ae^{-\alpha r^2}$. In the second the neutron-proton interaction operator depends on spin orientation as suggested by Wigner to explain the large scattering cross section of slow neutrons in hydrogen.[1] The interaction between like particles is represented by the potential $A_s e^{-\alpha r^2}$. Both A and A_s are determined as functions of α by requiring that the computed normal state energy, using a Gaussian-type wave function, equals the experimental binding energy of the alpha-particle. The function

$$\psi_{2p} = N(x_1+x_2-x_3-x_4)\exp\left[-\tfrac{1}{2}\nu(\pi_{13}^2+\pi_{14}^2+\pi_{23}^2+\pi_{24}^2) - \tfrac{1}{2}\mu(\pi_{12}^2+\pi_{34}^2)\right]$$

appears to be suitable for the study of the singlet $2p$ state of the alpha-particle with about the same degree of accuracy as the simple Gaussian-type function for the $1s$ state. With this wave function and the second Hamiltonian the $2p$ energy eigenvalue is $-18mc^2(1/\alpha^{\frac{1}{2}} = 2.4\times10^{-13}$ cm), $-13mc^2(1/\alpha^{\frac{1}{2}} = 2.2\times10^{-13}$ cm), $-8mc^2(1/\alpha^{\frac{1}{2}} = 2.0\times10^{-13}$ cm). With the first Hamiltonian the energies run about $7mc^2$ higher. The second Hamiltonian yields a singlet $2p$ state which is stable against spontaneous disintegration into H^3 and H^1 for $1/\alpha^{\frac{1}{2}} \geqq 2.3\times10^{-13}$ cm. Similar calculations on the triplet $2p$ state indicate that the triplet level lies below the singlet.

[1] Feenberg and Knipp, Phys. Rev. **48**, 906 (1935).

20. A Generalization of the Uncertainty Principle. BORIS PODOLSKY, *University of Cincinnati.*—The paper considers the knowledge of system α that may be obtained as a result of measurements performed upon another system β. A synthesis is proposed preserving the best aspects of the various points of view. It preserves the criterion of reality of Einstein, Podolsky and Rosen, but permits the conclu-

sion that quantum-mechanical description of physical reality is complete. In achieving this synthesis one is led to a generalization of the uncertainty principle which may be stated as follows: *If ΔQ and ΔP are the uncertainties in the values at any moment of time of a coordinate and its conjugate momentum respectively, which may be obtained by processes* (not necessarily simultaneous or even compatible) *each of which does not disturb the system under consideration, then $\Delta P \Delta Q > k$*, in the same way as in Heisenberg's uncertainty relation.

21. Ultraviolet Absorption Spectrum of Carbon Disulphide Vapor. J. RUD NIELSEN, F. W. CRAWFORD AND L. D. HUFF, *University of Oklahoma.*—A large number of absorption bands have been measured between 2300A and 1900A, and the bands between 3800A and 2900A have been remeasured. The variation of the intensities with temperature has been studied for both band systems by the method described by Clements.[1] Applying the results obtained from the Raman and the infrared absorption spectra, we have made some progress in the analysis of the band systems, although the intensity data have not yet been fully utilized.

[1] Clements, Phys. Rev. **47**, 220 (1934).

22. Haze and Smoke Visibility—Theory and Measurement. G. D. SHALLENBERGER AND E. M. LITTLE, *State University of Montana, Missoula.*—The present workers in collaboration with the U. S. Forest Service made a study of the visibility of smokes and of distant mountain ridges. This study included a mathematical treatment, based on the scattering of light by particles producing haze, Fechner's law of stimulus-sensation for vision, and visual resolving power. It also included measurements made in the field with a visibility meter that was originated in the course of the study, and which can be used also in aviation, etc. The visibility of a smoke is the distance it is from an observer when it can just be seen. A similar definition was used for ridge visibility. The visibility of a smoke in sunlight differs from that of a smoke in the shade. The azimuthal curves for the former are circles except on extremely clear days, while those of the latter have depressions in the direction of the sun when the sky is clear. With an overcast sky the shade smoke azimuthal curves are also circles. Under some conditions it was found that adding haze to the air *increases* the visibility. Binoculars increase visibility but not proportional to their power. Also, field and laboratory measurements were taken.

23. Simultaneous Sets of Interference Fringes. I. WALERSTEIN AND R. A. WOODSON, *Purdue University.*—In the usual arrangement of the Michelson interferometer a set of circular fringes is obtained if the dividing plate is half-silvered and the planes of the two mirrors are exactly perpendicular to each other. If the half-silvering is removed from the dividing plate it is possible to obtain, because of internal reflections, two nearly equally intense reflected beams and two transmitted beams, one much stronger than the other. A similar division of each incident beam takes place at the compensating plate. The various beams upon recombining at the dividing plate give rise simultaneously to at least three sets of visible fringes of different forms. Calculations of the path difference for any two interfering rays due to one ray passing through a number of glass plates and the other one passing through a lesser number of plates lead to a conic section form of the fringes. The shapes of the simultaneous sets of fringes change continuously as the difference in distances of the mirrors from the dividing plate, the angle between the dividing and compensating plate, and the angles between plates and mirrors is varied. They are circles, ellipses, straight lines, and double sets of hyperbolas, one with a horizontal and the other with a vertical axis but with common asymptotes.

24. Low Intensity Solarization for Three Emulsions. HENRY P. GRIGGS, S.J., *Saint Louis University.* (*Introduced by F. E. Poindexter.*)—The work of Reardon and Griggs[1] on Hammer Extra-Fast emulsions has been repeated (at Spring Hill College, Mobile, Alabama) for the Extra-Fast emulsions and also carried out with Hammer Ultra-Rapid Press and Special-Ortho Plates. The numbers of reversals obtained for a maximum exposure time of 11,854 minutes for the different emulsions were as follows: for the Extra-Fast, four; for the Ultra-Rapid, six; and the Special-Ortho, seven. The density increases at successive maxima in all cases except for the last maximum of the Special-Ortho emulsion which is slightly less than the preceding maximum density. With the exception of this one maximum, there is no damping down of the swings in the density *vs.* log E graphs, and hence the results do not agree with those of Lubovich on exposures at high intensities.[2]

[1] Reardon and Griggs, J. Opt. Soc. Am. **24**, 331 (1934).
[2] Lubovich, J. Opt. Soc. Am. **14**, 309 (1927) and Rev. Sci. Inst.

25. Pressure Effect and Physical Development. ANNA JOYCE REARDON, *Saint Louis University.* (*Introduced by F. E. Poindexter.*)—The effect of pressure on the latent photographic image was investigated with reference to three methods of physical development. These methods were: (1) Physical development before fixation, as described by Odell;[1] (2) pre-fixing with acid hypo and then development of the pressure-affected latent image in Odell's silver-nitrate-amidol developer; (3) pre-fixing of latent image in 30 percent plain hypo and development in Odell's developer. All three types of physical development show a reduction in latent image density. The amount of reduction depends upon the pressure applied and the type of development employed. The actual density obtained with physical development was in each case less than that obtained with chemical development, although the exposure time, for physical development, was twice as great. The density-log E graphs obtained when the underlying Br was dissolved out with 30 percent plain hypo were straight lines. The percent of density reduction for an exposure-time of 256 minutes and a pressure of 1357 kg/cm^2 was 13 percent for chemical development; 28 percent for physical development method (1); 41 percent for physical development method (2); and 68 percent for physical development method (3).

[1] Odell, Ind. Eng. Chem. **25**, 877–879 (1933).

26. A New Pressure Effect in Photography. FRANKLIN E. POINDEXTER, ANNA JOYCE REARDON AND ONA K. DEFOE, *Saint Louis University.*—We find that the application of hydrostatic pressure to a photographic film after exposure but before development lowers the developable image density. Eastman's Par-speed nitrate Portrait films were used. After exposure, the films were cut into two parts. One of these portions was placed in water at room temperature, the other part was sealed into a hard rubber vessel in water and placed in a high pressure apparatus. The two parts were chemically developed simultaneously. The pressures varied from 26,000 to 36,000 lbs./sq. in., and were applied from 20 to 30 minutes. We find that the density is lowered some 4 or 5 percent for an image density in the region of 0.7.

27. The Optical Constants of Liquid Sodium Amalgams. A. P. FRIESEN, *University of Kansas.*—The optical constants of liquid sodium and of 44 liquid sodium amalgams, between the limits of 0 and 40 atomic percent mercury, were determined. Large changes in n^2-k^2 and in $2nk$ with change in concentration occurred in the neighborhood of those concentrations where so-called intermetallic compounds exist. Incidentally more or less periodic small changes in n^2-k^2 and in $2nk$ were discovered at those concentrations where compounds are not known to exist. Work is in progress to obtain further experimental data that may be useful in the interpretation of these small periodic changes or "ripples."

28. A Mercury Manometer with High Multiplication Factor for Differential Pressure Measurements. RICHARD M. SUTTON, *Haverford College, Haverford, Pa.*—The maximum gain of deflection of liquid in one tube of an ordinary U-tube manometer approaches a limiting factor 2 obtained by making the ratio of areas of the side arms large. However, by using two liquids of different densities and by making a simple modification of design, it is possible to secure a real multiplication factor much greater. An ordinary U-tube of large bore is constricted in one arm at some point *above* the bend in the U. Mercury is inserted in the U to a level somewhat below this constriction. A light liquid is inserted above the mercury to fill the remaining volume of large tube and to extend into the smaller tube. For a given pressure difference, the light liquid in the smaller tube is now deflected farther than the mercury in a simple manometer by a factor $[2\rho_2 a/b]/[2\rho_2+(a/b-1)\rho_1]$ where a and b are the areas of the larger and smaller tubes, ρ_1 and ρ_2 are the densities of the light and heavy liquids, respectively. For maximum multiplication the two side arms containing mercury are made equal in area and *all* of the reduction in area occurs above the interface of the two liquids. This manometer combines the compactness of a mercury manometer with the sensitivity over any chosen pressure range of a light-liquid manometer. Application of the design to a sensitive acceleration meter and to a xylene-mercury barometer giving large deflections for normal atmospheric variations is described.

29. Surface Tension of Mercury in the Presence of Nitrogen, Hydrogen, and Carbon Dioxide. MARIE KERNAGHAN, R.S.C.J., *Maryville College of Saint Louis University.*—The apparatus and the method are those previously described by the author.[1] The combination of drying tubes and pressure gauge dispenses with stopcocks and is so constructed as to admit any gas in any quantity and to hold the pressure constant through any length of time. Nitrogen was obtained directly by passing air through pyrogallic acid solution; hydrogen and carbon dioxide were each prepared in a Kipp generator. The surface tension of mercury in a vacuum was again checked and found to be 476.5 dynes per centimeter at 20°C. The results obtained in the presence of the gases may be summarized as follows:

P (mm)	H_2	N_2	CO_2
<0.00001	475.5	476.5	476.5
.5	474.0	470.6	469.3
1	473.3	470.0	468.8
3	472.3	469.4	467.3
5	471.5	469.0	465.8
7	471.0	469.0	465.2
300	471.0	469.0	465.2

[1] Kernaghan, Phys. Rev. **47**, 202A (1935).

30. Variation with Altitude of the Optimum Thickness of Lead for the Production of Cosmic-Ray Showers.—J. C. STEARNS, *University of Denver,* AND DAROL K. FROMAN, *Macdonald College, McGill University.*—The number of showers obtained per minute from various thicknesses of lead scattering blocks were counted by a triple-coincidence Geiger counter system at altitudes ranging from sea level to 14,120 ft. Two methods were used. In the first the three counters were arranged in the form of a triangle and counts were taken with various thicknesses of lead above the top counter. In agreement with B. Rossi,[1] the thickness of lead giving the maximum number of showers is found to be 1.7 cm and within experimental error, to be independent of altitude. In the second method the three counters were placed in a vertical line and the difference in counting rate, with the lead scatterer placed firstly above the top counter, and secondly between the two lower counters, was taken to be a measure of the number of showers produced. This method gave a much sharper maximum at 1 cm of lead.

[1] Rossi, Report of International Conference on Physics, Vol. I, p. 245.

31. The Variation with Altitude of the Absorption of Cosmic and Shower Radiation. DAROL K. FROMAN, *Macdonald College, McGill University,* AND J. C. STEARNS, *University of Denver.*—The absorption in lead, zinc and aluminium of primary cosmic rays has been measured by a triple Geiger counter at altitudes ranging from sea level to 14,120 ft. The absorbing material was placed between the counters, a lead block two inches thick was kept between the counters to absorb any extremely soft radiation. For thicknesses of material less than 16 cm some of the average absorption coefficients are given in the table. The absorption

Altitude (ft.)						
120	5,300	10,800	5,300	10,800	5,300	10,800
Material						
Pb	Pb	Pb	Zn	Zn	Al	Al
Absorption coefficient (cm^{-1})						
0.014	0.018	0.023	0.012	0.018	0.006	0.010

of the shower radiation was measured by using three counters in triangle formation. A scattering block of lead was placed above the system and the absorption measured in lead sheets placed just above the lowest counter. The apparent absorption coefficient of the showers from lead varies from about 0.5 cm^{-1} Pb at sea level to about 0.4 cm^{-1} Pb at 14,120 ft. elevation. The absorption in lead is not quite exponential and the shape of the absorption curves indicates the possibility that shower particles may have the ability to excite new showers.

32. Fluctuations in Cosmic-Ray Ionization as Given by Several Recording Meters Located at the Same Station. Richard L. Doan, *University of Chicago.*—A graphical comparison will be given of the intensity variations of cosmic rays as registered on several different recording meters during a period of ten weeks. As one might expect, the agreement is statistical rather than coincident. The barometer effect as calculated from the data of the individual meters ranges from 1 to 2 percent per cm of mercury. A diurnal analysis carried out for a ten-day period during which barometric changes were small indicates a maximum of intensity at about 9:00 A.M. having a magnitude 0.19 percent greater than the average. The average number of cosmic-ray ionizing particles traversing unit area of the ionization chamber per unit time, as estimated from statistical fluctuation data, is shown to compare favorably with counter observations. As regards ionization bursts: Analysis of distribution-in-time of the bursts shows that they occur in a purely random manner. Size-frequency distributions can be represented for the most part by a single exponential function. Analysis of nearly 700 bursts $>15(10)^6$ ions occurring over a 25 mm Hg range of atmospheric pressure yields no evidence for a barometer effect on burst frequency although the consistency of the data is not sufficient to rule out the possibility of an effect as large as 8 percent per cm Hg.

33. Heavy Particles Produced by Cosmic-Ray Encounters. Carl D. Anderson and Seth H. Neddermeyer, *California Institute of Technology.*—Our preceding measurements on the energies of cosmic-ray tracks made at sea level have revealed but a few cases in which heavy particles appear as a result of cosmic-ray encounters. Similar measurements made this summer on top of Pike's Peak bring to light unambiguously a considerable number of such particles—approximately 100 out of 7000 successful counter-tripped exposures. These tracks possess the following properties: (1) In the great majority of cases these tracks seem to be made by protons. (2) They appear to be ejected in all directions and to have little relation to the direction of the incoming beam. (3) In several cases protons and electrons originate in the same center, the energies of the latter being sometimes more than 100 MEV. (4) These proton tracks appear as the result of disintegrations produced both by ionizing and nonionizing rays. (5) The high values of the energies of the disintegration particles preclude the possibility that the disintegrating nucleus could have supplied this energy. The foregoing results, on the other hand, show that both ionizing and nonionizing rays may be so absorbed as to appear in part in the ejection of heavy particles from a nucleus. (6) Practically all the heavy tracks can be interpreted only as secondaries produced within the atmosphere or the material above the chamber. (7) Certain types of disintegrations, heretofore unobserved at sea level, indicate the presence of neutrons at high altitudes. It is a pleasure to acknowledge the aid given to these researches by the Carnegie Institution of Washington and the Union Oil Company of California.

34. Gamma-Ray Ionization Currents in Air. James W. Broxon and George T. Merideth, *University of Colorado.*—An ionization chamber particularly adapted to measurements over rather wide ranges of temperature, pressure, intensity of the collecting field, and intensity of ionizing radiation has been constructed. Entire uniformity of collecting field has not been provided, but in the main volume of the ionized gas there is not great variation from uniformity. Measurements of currents due to ionization by a weak gamma-ray source combined with natural ionization have been made in air at room temperature. With gradients extending to about 4000 volts/cm, currents have been measured at a few pressures between local atmospheric pressure and 170 atmospheres. At the higher pressures, the current continues to increase with the strength of the collecting field throughout the range employed. Except with very low gradients, the rate of accumulation of negative ions is greater than that of positives (see recent observations of Clay and Van Tijn[1]). With the high gradients, the current-pressure relations appear roughly to resemble those observed by one[2] of us with only the natural ionization and much lower gradients. It is expected that further data will be available for the meeting.

[1] Clay and Van Tijn, Physica 2, **8**, 825 (1935).
[2] James W. Broxon, Phys. Rev. **37**, 1325 (1931).

35. The Yield of Alpha-Particles from Li^7 Bombarded by Slow Protons. H. D. Doolittle, *University of Chicago.* (*Introduced by A. J. Dempster.*)—The absolute yield of alpha-particles from a thick target of lithium bombarded by protons of energies from 23 to 72 electron kilovolts has been determined. The logarithm of the number of disintegrations per proton has been plotted against $V^{-\frac{1}{2}}$ where V is the energy of the proton in electron volts. The results of Herb, Parkinson and Kerst as well as those of Oliphant and Rutherford have been included in the graph to extend it over the range from 23 to 400 kilovolts. This curve is not a straight line as expected by the theory but is slightly concave upwards. No indications of resonance nor of a minimum voltage required for disintegration were observed. An FP-54 Pliotron in a modified form of the DuBridge-Brown balanced circuit was used in photographic recording of the alpha-particles.

36. The Energy Levels of the Nuclei of Light Elements. H. A. Wilson, *Rice Institute.*—Regarding atomic nuclei as combinations of neutrons and protons their energies of

formation may be supposed due to the formation of bonds analogous to the chemical bonds in for example hydrocarbons. The energies of formation may then be explained by assuming suitable constitutions and bond energies. It is found that the energies of formation and the nuclear reaction energies of several of the light elements may be explained in this way. In this paper it is shown that the energy levels of several light elements may be explained by supposing some of the bonds normally present to be missing.

37. Theory and Operation of an "Achromatic" Mass Spectrograph. A. J. DEMPSTER, *University of Chicago.*—A mass spectrograph has been constructed which brings a divergent bundle of charged atoms to a focus even when different velocities are present. This analog to the achromatic doublet lens first separates the various velocities by an electrostatic field, and reunites them by means of a magnetic field. Even with comparatively wide slits an increased resolving power is obtained. With a high frequency spark as a source the isotopes of elements not obtainable by other methods have been observed. Mass spectra of gold, palladium, rhodium, platinum, uranium, and iridium have been made. Preliminary results with nickel, cadmium, tin, cerium, and lead will be reported, and the behavior of the instrument illustrated by reproductions of spectra.

38. The Existence of Isobares of Adjacent Elements and the Mass of the Neutrino. KENNETH T. BAINBRIDGE AND EDWARD B. JORDAN, National Research Fellow, *Harvard University.*—Mattauch and Sitte have suggested that isobares differing by one unit in atomic number either are unstable or do not exist at all. Beck, Bethe and Heisenberg have shown that isobares of this class may be accounted for on the neutrino hypothesis for a neutrino mass $m_n > 0$. Germanium, cadmium, indium, tin, tellurium, mercury, lead, and bismuth have been analyzed with a linear scale mass spectrograph which combines high sensitivity, resolving power, and dispersion. Spectra of singly and doubly ionized isotopes from hydrogen free sources give definite proof of the existence of three isobaric pairs, Cd^{113} In^{113}, In^{115} Sn^{115}, and Sb^{123} Te^{123}, which apparently are stable. Both members of each of these pairs could be stable for $m_n > 0$. Alternatively the members of higher atomic number could change by K-electron capture with or without benefit of a neutrino, these transmutations being beyond the possibility of detection by the experimental tests applied to these elements. β-instability is manifest for all other definitely attested cases of adjacent isobares with the exception of the proton-neutron, H^3He^3 pairs. The isotopes Cd^{115}, Sn^{121}, Hg^{197}, and Pb^{209}, which are members of isobaric pairs, cannot be present to greater than 1/8, 1/60, 1/2, 1/10, respectively, of the abundance claimed.

39. The Spectrum of Th IV.* R. J. LANG, *University of Alberta, Edmonton.*—The spectrum of thorium has been photographed from 3000A to 350A with the vacuum spark and the spark in air and in nitrogen as sources. The following multiplets have been located for the trebly-ionized atom: $7S-7P$, $6D-7P$, $7P-7D$, $7P-8S$, $7P-8D$, $7P-9S$, $6D-5F$, $5F-8D$, $5F-5G$.

* To be read by title.

40. Fine Structure of α-Lines of Hydrogen and Deuterium. R. C. WILLIAMS AND R. C. GIBBS, *Cornell University.*—A further detailed study of the fine structure of the $H\alpha$- and $D\alpha$-lines has been made by using a discharge in hydrogen gas and in deuterium gas each of extremely high purity, the latter having been kindly furnished by Professor H. C. Urey. At least four photographs of the interference fringe pattern, obtained from the discharge in each of the gases at three substantially different pressures and at two different densities of the discharge current for each of these pressures, were made. While the measured peak to peak "doublet" interval, at least six values of which could be obtained from each plate, varied with the discharge conditions, remarkably uniform values for this interval were obtained from the plates taken under any given discharge condition. However, the diverse interval values obtained under different discharge conditions yielded practically the same interval between any two of the resolved components when suitable account was taken in each case of the shift in peak position due to overlapping components. The average interval between the two major components was found to be nearly the same for both hydrogen and deuterium but about 0.01 cm^{-1} less than that predicted by theory. In the case of deuterium the minor component nearest to the high frequency major component was clearly resolved from the latter. However, the interval between these components was found to be 0.134 cm^{-1} in contrast with that of 0.108 cm^{-1} obtained from theory. An equally large separation of these components was found to be consistent with the analysis of the intensity complex obtained for $H\alpha$. This large deviation from theory may possibly be due to a Stark effect produced by ionic electric fields in the discharge.

41. The e/m Ratio as Determined from the Interval Between Corresponding Components of $H\alpha$- and $D\alpha$-Lines. R. C. GIBBS AND R. C. WILLIAMS, *Cornell University.*—By using a mixture of hydrogen and deuterium such as to yield spectral lines of equal intensity from the two gases, and an interferometer spacing of such a size as to bring the interference fringes from one gas about half-way between those from the other gas and at the same time prevent overlapping of neighboring fringes, the ratio of e/m has been redetermined from intervals between corresponding components of the $H\alpha$- and $D\alpha$-lines. Several measurements of such intervals were obtained from each of five plates. After suitably correcting for shifts due to overlapping components and reducing the interval to vacuum conditions an average value of $\Delta\nu_{H-D} = 4.1470 \pm 0.0004$ cm^{-1} was obtained for the interval between the $2p\,^2P_{3/2}-3d\,^2D_{3/2}$ components. Upon combining the above interval value with certain atomic data, in a manner heretofore reported, wherein the atomic weights of the proton and deuteron as recently determined by Bethe from disintegration data were used and the value of the Faraday was adjusted to conform with Aston's scale of atomic weights such that $O^{16} = 16$, the specific charge of the electron was determined as $e/m = 1.7577 \pm 0.0004 \times 10^7$ e.m.u./gram and its atomic weight as $5.4908 \pm 0.0005 \times 10^{-4}$.

42. The Paschen-Back Effect. III. $^2S^2P$ Multiplets in Strong Fields. J. B. GREEN AND R. A. LORING, *Ohio State University and the University of Louisville.*—The Paschen-Back effect of $^2S^2P$ multiplets has been discussed and the theory verified in detail. In particular, the $^2S^2P$ multiplet of Li, λ6708, has been resolved in reversal and shows the fine structure predicted but not heretofore observed.

43. The Paschen-Back Effect. IV. Mutual Spin-Orbit Interaction in Two-Electron Spectra. R. A. LORING AND J. B. GREEN, *University of Louisville and Ohio State University.*—The effect of a strong magnetic field on the Be triplet λλ3321, $^3P^3S$ has been studied. The experimental patterns have been compared with calculations which take account of the interaction between the spin of one electron and the orbit of the other. The experimental results are in excellent agreement with the theory, but in some disagreement with Back's work on this group of lines.

44. Forgotten Terms in the Isotope Effect in Molecular Spectra. J. H. VAN VLECK, *Harvard University.*—If molecular energy levels are represented by an empirical formula $W=W_0+\hbar\omega(v+\frac{1}{2})+BJ(J+1)+\cdots$, the observed ratios $\rho_v=\omega_1/\omega_2$ and $\rho_r=B_1^{\frac{1}{2}}/B_2^{\frac{1}{2}}$ usually do not quite equal the square root of the ratio μ_2/μ_1 of the reduced masses for the two isotopes. Strict equality is characteristic only of the ideal rotator-vibrator of infinitesimal amplitude, and correction formulas due to various causes have been given by Dunham and Crawford, Kronig, and Mulliken. Even when their results are combined, we find that many correction terms of magnitude comparable with theirs have been omitted, some being of the order $B^2/\hbar^2\omega^2$, others of the order $B/h\nu$ relative to unity, where ν is a typical electronic frequency. When all terms are included, there are so many constants that exact theoretical computation of ρ_v and ρ_r would be very difficult. ρ_v and ρ_r should not be exactly equal, and, at least for the ground state, should ordinarily both be slightly nearer unity than is the ideal value $(\mu_2/\mu_1)^{\frac{1}{2}}$. New terms are also found in the isotope shift in the constant term W_0.

45. Excitation of Sodium Hyperfine Structure in a Molecular Beam. BENJAMIN CARPENTER AND RUSSELL A. FISHER, *Northwestern University.*—A spectroscopic source suitable for hyperfine structure investigations has been constructed employing a molecular beam excited by high frequency oscillations in argon, the argon being at such a pressure that the beam is not destroyed by scattering. When sodium is excited in the apparatus the Doppler broadening is reduced such that the half-intensity breadths of the components of the D lines are less than 0.011 cm^{-1}. A consideration of the Doppler broadening in such an apparatus indicates that the half-intensity breadth to be expected if the exciting encounters are entirely with electrons is about half that observed, while if the encounters are with argon atoms or ions the broadening should be three times the observed value. Observations upon the hyperfine structure of the D lines by means of a Fabry-Perot interferometer with spacers of 48 mm and 110 mm yielded no structure in addition to that already observed by others. Our observed separation of the two components of D_1 is 0.062 cm^{-1} and of D_2 is 0.057 cm^{-1}. Accepting the value of $I=3/2$ for sodium and the theoretical ratio of 5 : 1 for the splitting factors of the states $3p\,^2P_{\frac{1}{2}}$ and $3p\,^2P_{1\frac{1}{2}}$ we obtain by graphical analysis that the splitting factor for the state $3s\,^2S$ is 0.030 cm^{-1} and for $3p\,^2P_{\frac{1}{2}}$ is 0.004 cm^{-1}.

46. Quenching and Depolarizing Collisions and Polarization of Resonance Radiation in a Magnetic Field. RAYMOND PETERSEN* AND A. ELLETT, *State University of Iowa.*—On assuming that excited atoms may leave the initial excited state (1) by radiation, (2) by a collision of the second kind, (3) by a depolarizing collision (i.e., a collision which merely destroys coherence), we may write for exciting light polarized along x, observation along z in a magnetic field parallel to z, in the absence of hyperfine structure,

$$\frac{I_x-I_y}{I_x+I_y}=R=R_0\left\{[1+4(\omega\tau^*)^2]\left[1+\frac{(3-R_0)\alpha'p\tau}{3(1+\alpha p\tau)}\right]\right\}^{-1},$$

where α, α' are quenching and depolarizing efficiencies and $1/\tau^*=1/\tau+(\alpha+\alpha')p$. The extension to include hyperfine structure is obvious. A corresponding expression for the rotation of the plane of maximum intensity is also readily obtained. Observed and computed values of R for the mercury λ2537A line in the presence of hydrogen and for the angle of rotation of the plane of maximum intensity are in satisfactory agreement using $\alpha=1.39\times10^7$, $\alpha'=1.81\times10^7$ collisions per excited atom per sec. per mm Hg hydrogen pressure.

* Deceased July 25, 1935.

47. Design of an Ionization Gauge for Atomic Beam Measurements. R. D. HUNTOON AND A. ELLETT, *State University of Iowa.*—Use of two types of current measuring devices, galvanometer and vacuum tube amplifier, with an ionization gauge shows that the stable sensitivity,

$$\left(\frac{\text{positive ion current/mm gas pressure}}{\text{current fluctuation}}\right),$$

is greater for low electron emission densities thus favoring the use of the amplifier with its greater current sensitivity. Such factors as electrode size, relative potentials, electron emission, interelectrode leakage, type of inlet, gas adsorption, surface charges, temperature changes, B.K. oscillations, and variations of pumping speed which influence the design of a gauge are discussed and an arrangement to fulfill the necessary requirements is described. Performance tests show the gauge to be capable of measuring beam pressure changes at least as small as 3×10^{-11} mm of mercury or approximately 3×10^9 mercury atoms per square centimeter per second.

48. Intensity of Scattering of Electrons as a Function of Angle. H. J. YEARIAN AND J. D. HOWE, *Purdue University.*—Recent experiments on the scattering power (atom form factor) of Au and Ag for high velocity electrons[1] have shown that the scattering power deviates from that

given by the usual Born approximation when the electron energy is comparable with, or lower than the K excitation potential of the scattering atom. Since the diffraction from these cubic substances gave a small number of points on the scattering curves, the experiments have been continued using hexagonal Cd as a scatterer. The curves obtained from Cd (48) are similar to those from Ag (47) but the larger number of experimental points in the case of Cd show a greater and more complex deviation from the simple theory than do the experiments with Ag.

[1] H. J. Yearian and J. D. Howe, Phys. Rev. **48**, 381 (1935).

49. X-Ray Scattering in Molten Salts. I. K. Larkhorovitz and E. P. Miller, *Purdue University.*—Diffraction patterns from molten salts have been obtained with monochromatic silver $K\alpha$ radiation. The liquid salt films of about 0.5 mm thickness are held in a hot platinum wire loop at a temperature close to the melting point. LiCl, LiBr, NaCl and KCl have been investigated so far: The diffraction patterns of LiCl, LiBr are sharper than the patterns usually observed in liquids. NaCl yields a diffuse diffraction pattern as contrasted with KCl, the pattern of which is much sharper. This can be understood by taking into account the influence of the different diameter and scattering power of the ions. The diffraction pattern of KCl has been analyzed under the assumption that the scattering power of K and Cl is approximately the same: *In the neighborhood of any scattering center there is a concentration of ions in a distance of about 3.3A and again at about 4.3A;* in the KCl crystal any atom has 6 equivalent neighbors in a distance of 3.14A and 12 equivalent neighbors in a distance of 4.44A.

50. Further Comparison by X-Ray Diffraction of Para-azoxyanisol in Liquid Crystalline and Liquid Phases. G. W. Stewart, *State University of Iowa.*—Additional experiments have emphasized the almost insuperable difficulties in securing a diffraction intensity distribution without introducing an error caused by convection. The conclusions are (a) that the structure of the liquid crystalline phase is the more orderly, (b) that the two structures are probably similar, differing also in stability, (c) that the liquid crystalline phase may be regarded as produced by aggregations of cybotactic liquid "groups" plus the above indicated changes, and (d) that a temperature gradient in the liquid crystal phase produces an effect equivalent to the preferential orientation of the swarms perpendicular to the direction of the heat flow.

51. Pseudo-Cubic Lattices and the Principle of "Variate Atom Equipoints." F. C. Blake, *Ohio State University.*—Pseudo-cubic lattices perhaps could be said to be of two sorts, those having angles slightly different from right angles and those having right angles in the tetragonal system but not completely equivalent to the cubic lattices. By an intensive study of both simple tetragonal and body-centered tetragonal lattices at the axial ratio $\sqrt{2}$ the parameter flexibility is greatly increased and it is in all probability possible to explain all of the intensity variations in the "spinels" without invoking the aid of the principle of "variate atom equipoints." An illustration is zinc stannate, kindly supplied by the staff of the Geophysical Laboratory at the Carnegie Institution. It can be definitely assigned to space group C_{4v}^{11}. This naturally raises the question for the need of this principle, at least in the "spinel" problem.

52. Theory of the Effect of Temperature on the Reflection of X-Rays by Crystals. I. Isotropic Crystals. G. E. M. Jauncey and C. Zener, *Washington University.*—The Debye-Waller temperature factor e^{-M} in $F=fe^{-M}$ is simply and briefly derived as follows: For a vibrating string the mean square displacement μ_r due to the rth vibrational mode of frequency ν_r is related to the energy W_r in this mode by $\mu_r=W_r/4\pi^2\nu_r^2m$, where m is the mass of the string. For a solid we have a similar formula but now μ_r is in any direction. In the z direction the relation for the mean square displacement is $\overline{z_r^2}=W_r/12\pi^2\nu_r^2m$, where m is the mass of the solid. In Debye's specific heat theory the thermal agitations are the result of adding many vibrational modes up to a limiting frequency ν_m. Since the phase relations between these modes are random the resultant $\overline{z^2}=\Sigma\overline{z_r^2}$. The number of modes in the range $d\nu$ is $dr=9n\nu^2d\nu/\nu_m^3$, where n is the number of atoms in the solid. Also in specific heat theory $W_r=h\nu_r/(e^{h\nu_r/kT}-1)+h\nu_r/2$. We now integrate and obtain $\overline{z^2}$. It is known that the Debye-Waller $M=8\pi^2\overline{z^2}(\sin^2\theta)/\lambda^2$. Putting in the value of $\overline{z^2}$ just found, we obtain Waller's formula for M.

53. Theory of the Effect of Temperature on the Reflection of X-Rays by Crystals. II. Anisotropic Crystals. C. Zener, *Washington University.*—The method discussed in the previous paper is extended to include the general case of anisotropic solids. Since M is a quadratic function of the direction cosines of the normal to the reflection plane, M is completely determined by measurements from three mutually orthogonal reflection planes. For crystals with three-, four-, or sixfold axes of symmetry, M will be a function only of the angle Θ between the principal axis and the normal to the reflection plane. The function M has been determined for two metals with a sixfold axis of symmetry, Zn and Cd. The results are

$$M_{Zn}=1.1\times10^{-3}\{1.80\cos^2\Theta+\sin^2\Theta\}T\sin^2\theta/\lambda^2,$$
$$M_{Cd}=1.6\times10^{-3}\{1.73\cos^2\Theta+\sin^2\Theta\}T\sin^2\theta/\lambda^2.$$

The calculation is complicated by two factors. Firstly, for each direction of propagation there are three different wave velocities associated with a linearly polarized plane wave. Secondly, the vibration associated with each velocity is not either parallel or normal to the direction of propagation, corresponding to the longitudinal and transverse waves of isotropic solids, but forms an oblique angle with this direction.

54. Dependence of Diffuse Scattering of X-Rays from Quartz Upon the Angle Between the Crystal Axis and the Plane of Scattering. W. A. Bruce and G. E. M. Jauncey, *Washington University.*—According to Woo and Jauncey's theory the coherent part of the x-rays diffusely scattered from a crystal is proportional to $(1-e^{-2M})$, where M is the Debye-Waller temperature factor. For quartz, Young's modulus is 10.1 and 7.7×10^{11} dynes/cm^2 for directions

respectively parallel and perpendicular to the axis of the crystal. According to the previous paper by Zener this difference in the moduli should give different values of M for elastic vibrations parallel and perpendicular to the axis—in other words, different characteristic temperatures should be associated with the different directions. Rough calculations give 550 and 485°K, respectively. X-rays of average wave-length 0.5A were scattered from an X cut plate of quartz at 20° scattering angle for various settings of the axis relative to the plane of scattering. We assured ourselves of the absence of Laue spots by subjecting the crystal to piezoelectric oscillations which Jauncey and Deming have shown do not affect the diffuse scattering. Theory indicated a change of about 9 percent from axis perpendicular to axis parallel to the scattering plane. This order of change was found.

55. Properties of Prins' Formula for the Diffraction of X-Rays by a Perfect Crystal. ROY C. SPENCER, *University of Nebraska.*—Prins' formula for the intensity I may be reduced to the following forms. $I^{\frac{1}{2}}=r=|z-(z^2-1)^{\frac{1}{2}}|=a-(a^2-1)^{\frac{1}{2}}=(b^2+1)^{\frac{1}{2}}-b=a-b$ where z is the complex variable $(x-i\alpha)/(m+in)$, $m^2+n^2=1$, α and n depend on the absorption coefficients, and a and b are the semi-major and minor axes of the confocal ellipse which cuts the point x. (Its foci are at $x=\pm m$, $y=\alpha\pm n$.) *Graph.* Measure the average distance from the point x to the two foci. This is a. Substitute in $I=(a-(a^2-1)^{\frac{1}{2}})^2$ which is Darwin's formula for the case of no absorption. *Integration.* Change the variable from r to $b=(r^{-1}-r)/2$. Then

$$x=[-mn\alpha\pm b(b^2+1)^{\frac{1}{2}}(b^2+n^2-\alpha^2)^{\frac{1}{2}}]\div(b^2+n^2)$$

which enables the area, center of gravity and percent reflection to be obtained by elliptic integrals or approximated by simple expressions. *Effect on x-ray lines:* The effect of the crystal curve on the asymmetry of the x-ray line curve is discussed. Its effect in broadening the x-ray line curve was discussed by the author at the Minneapolis meeting.

56. Effect of Angle of Incidence Upon Relative Intensities of X-Ray Lines from Ruled Gratings. ELMER DERSHEM, *University of Chicago.*—Those who have worked with ruled gratings in the soft x-ray region know that the intensity of spectral lines varies with the glancing angle of incidence but the magnitude of the effects which may be caused by small changes of angle is perhaps not generally appreciated. For example using the inside order from a glass grating having 600 lines per mm the ratio of intensity of the $L\alpha_{1,2}$ to the L1 line of nickel changed from 3 : 1 at a glancing angle of 3° 0′ to a ratio of 1 : 3 at 3° 5′. The corresponding change in this ratio for the first outside order was about 50 percent. Spectrograms and photometer measures over the range of possible angles using the L spectra of iron and copper taken with three different gratings show similar results. In general the relative intensities of the longer wave-lengths are greater at the larger angles. These results show that the intensity of any particular line may be greatly enhanced by a proper choice of the angle of incidence thus permitting a better study of any property pertaining to that wave-length. Any measures of the relative intensities of lines from a ruled grating must, however, be interpreted with caution.

57. Relative Intensities of $K\alpha$ X-Ray Satellite Lines. LYMAN G. PARRATT, National Research Fellow, AND F. K. RICHTMYER, *Cornell University.*—With a two-crystal vacuum spectrometer the intensities of the $K\alpha$ satellite lines relative to the intensities of the $K\alpha_1$ diagram lines have been measured for elements S(16) to Ge(32). The satellite structure in the region of $K\alpha_{3,4}$ includes α', α_3, α_3', α_3'' (a new component) and α_4. For elements S(16) to Ti(22) an additional line is observed very close to $K\alpha_1$. The relative intensities of each of these satellites have also been measured. The ratios of the energy of the $K\alpha_{3,4}$ group (which does not include the satellite close to the α_1 line) to the energy of the $K\alpha_1$ line for the respective elements are (not corrected for the finite resolving power of the spectrometer)

Z	$(I_s/I\alpha_1)\times100$	Z	$(I_s/I\alpha_1)\times100$	Z	$(I_s/I\alpha_1)\times100$
16	6.90	23	1.75	27	0.73
19	3.71	24	1.34	28	0.61
20	3.20	25	1.08	29	0.53
21	2.65	26	0.89	30	0.41
22	2.10			32	0.28

The measurements are in qualitative agreement with the theoretical relative intensities as calculated (recent unpublished work of R. D. Richtmyer) on the assumption that the initial atomic state for satellite emission is one of KL ionization.

58. Variation of the Atomic Structure Factor of Potassium with Wave-Length in the Region of the Potassium K Absorption Limit. WILLIAM P. JESSE AND SAMUEL K. ALLISON, *University of Chicago.*—A vacuum ionization spectrometer has been constructed in which an externally controlled, motor driven slide brings either a briquet of powdered NaF or powdered KF over the axis of rotation and into the path of the x-ray beam. Comparisons of the diffracted intensities from NaF and KF have been made with an ionization chamber Pliotron circuit. A range of wave-lengths from 1.54A (Cu $K\alpha$) to 3.93A (Ag $L\beta_1$) has been used. In this range Sb $L\alpha_1$ (3.4318A) has been used as one of the wave-lengths. The wave-length of the potassium K absorption limit is given as 3.4277A. The potassium structure factors were computed, based on those of NaF, which were assumed invariant in this wave-length range. On the short wave-length side of the K limit, the expected decrease in the atom factor was observed, and the experiments show a remarkable agreement with Hönl's theory. The predicted invariance of the structure factor decrement with $(\sin\theta)/\lambda$ was experimentally confirmed. On the long wave-length side of the limit the structure factor rises with increasing wave-length but results higher than Hönl's theoretical values are always obtained. This agrees with results of Lameris and Prins on dispersion in nickel. A search for any systematic errors which may enter on the long wave-length side is being made.

59. Experiments on the Efficiencies of Production, and the Half-Lives, of Radio-Carbon and Radio-Nitrogen. SAMUEL K. ALLISON, *University of Chicago*, Fellow of the Guggenheim Foundation at Cambridge, England.—The experiments of Cockcroft, Gilbert, and Walton on artificial radioactivity produced by bombardment of boron and carbon with protons and deuterons have been continued. Targets of B_2O_3 which had been bombarded with deuterons at 520 kv were ignited in oxygen and the half-life of the resultant radio-carbon dioxide measured. This gave a reliable half-life of 21 ± 0.6 minutes, whereas previous measurements on the half-life of the solid target had given variable values from 18 to 25 minutes, due to various complicating factors. No evidence could be obtained for the production of radio-carbon from boron by 520 kv protons. Radio-nitrogen was prepared by bombardment of carbon by 520 kv protons and by 520 kv deuterons. Its period was found to be 10.3 ± 0.3 seconds. The efficiency of production of radio-nitrogen by protons of this energy was found to be 1 atom of radio-nitrogen for 1×10^{10} protons. The efficiency of production by deuterons of this same energy was one in 2.3×10^8. The bombarding beam had been magnetically analyzed, and the gas mixture used in the deuterium experiments was composed of deuterium (from 98 percent heavy water) and helium. The yields agree well with those given recently by Hafstad and Tuve at the same voltage.

60. Fluorescence of the Chlorophyll Series: Fluorescence and Photodecomposition of Solutions of Chlorophyll *a* under O_2, CO_2 and N_2. H. V. KNORR AND V. M. ALBERS, *Kettering Foundation, Antioch College.*—The fluorescence spectra of chlorophyll *a* in solution in anhydrous acetone, and benzene, under atmospheres of O_2, CO_2 and N_2, have been investigated by the method previously described.[1] The total radiation from Pyrex mercury arcs was used for excitation. In the acetone solutions, maxima on the densitometer curves were observed at 679, 647 mμ under O_2, 677, 646 mμ under CO_2, 679, 648 mμ under N_2. In benzene the corresponding maxima were located at 683, 651 mμ under O_2, 683, 650 mμ under CO_2, 679, 649 mμ under N_2. The appearance of the fluorescence spectra for the acetone solutions during the photodecomposition is quite different from that of the benzene solutions under the same atmospheres. Stable conditions are apparently established in the benzene solutions since the intensity of the fluorescence is quite strong after many hours of exposure to the radiation. The fluorescence in the acetone solutions under N_2 and CO_2 vanished after two hours exposure. The appearance of additional maxima, and variation of the relative intensities of the maxima, indicate that the solvent and atmosphere are some of the determining factors in the photodecomposition of chlorophyll *a*.

[1] H. V. Knorr and V. M. Albers, Phys. Rev. **43**, 379 (1933) (Cold Spring Harbor Symposia on Quantitative Biology, Vol. 3, 1935).

61. Extending the Range and Usefulness of the Zeleny Electroscope by Automatically and Mechanically Counting Rapid Oscillations. R. W. BOYDSTON, *Miami University.*—Several publications call attention to the usefulness of the Zeleny electroscope for demonstrations and other laboratory purposes. Some difficulties are inherent in the operation of this instrument. The leaf repeatedly sticks to the plate, and the current range is limited by the inability of the observer to accurately count rapid oscillations. The author's work shows how both of these drawbacks are overcome. The circuit used is both simple and positive in its action. Currents ranging from 10^{-12} to 10^{-8} ampere may be measured. The counting device is capable of recording leaf oscillations as high as 125 per second. This extends the range of the Zeleny to its maximum current measuring possibility and overlaps the range of an ordinary galvanometer. Data are given on applying this device to investigation of some electrical properties of paper. Attention is called to numerous other applications. The extensions discussed should make the Zeleny electroscope a useful and reliable research instrument. Demonstrations will accompany the paper.

62. A Copper Sulphide Photo-Cell.* JAMES J. BRADY AND W. F. SPRENGNETHER, JR., *Saint Louis University.*—A copper sulphide photovoltaic cell is described in which the amount of sulphur used to form the sulphide is controlled by means of a molecular beam. The sulphide is formed by depositing sulphur atoms on a copper plate heated by electronic bombardment in a high vacuum. A ground-glass joint permits the copper plate to be rotated through 90° in order that it may be brought in contact with a metallic grid through which the light passes to the sulphide surface. A tungsten filament lamp is used as a light source, and the photocurrent is measured with a galvanometer. With no light on the cell and while the plate is still warm, a current flows through the circuit but comes to zero after the plate cools. When the light falls on the surface, the current flows in the opposite direction. The magnitude of the photocurrent is found to be greatly dependent on the gas pressure within the cell. The sensitivity decreases by a factor of about 10 when the pressure is increased from 10^{-5} mm to atmospheric. The sensitivity is completely restored by pumping the cell down to a pressure of 10^{-5} mm.

* To be called for after paper No. 9.

63. Interference of Neutron Waves in Ortho- and Parahydrogen.* EDWARD TELLER, *George Washington University, Washington, D. C.*—The anomalously large cross section observed for the scattering of slow neutrons by protons can be explained according to Wigner by assuming a dependence of the scattering on the relative orientation of the proton and neutron spins. This hypothesis leads to an energy for the singlet state of the deuteron of about $\pm 50{,}000$ volts. (The signs refer to a virtual or stable state respectively.) Direct evidence for the influence of the spin could be obtained from observations on the scattering of a homogeneous, monochromatic beam of neutrons in H_2. If no dependence on the spin exists, then one should expect that the scattering of neutrons by the two protons of H_2 should give rise to interference effects in both ortho- and parahydrogen. Assuming, however, that scattering takes place only when the neutron and proton spins have opposite orientations, interference effects would only exist in ortho-hydrogen. Actually the scattering for parallel spins should be small but not negligible. From the way in which

the phases of the neutron waves scattered by the two protons of para-H_2 are superimposed one can decide whether the singlet state of the deuteron is stable or virtual.

* To be called for after paper No. 20.

64. I. The Propagation of a Shock-Wave in Air. L. THOMPSON, *Naval Proving Grounds, Dahlgren, Va.*—From a consideration of conditions at the boundary at the instant of "explosion," and at great distances, a formula of the Riemann type has been derived for the velocity of propagation of the finite-condensation pulse, which fits the data obtained in the experiments of Part II throughout the field of observation. A corresponding formula for reduced times is given which represents in general the wave displacement (in half-diameters from the source), regardless of scale. Introducing the condition that the velocity at the surface of the explosive charge is V_d (the velocity of the detonating wave) the formula is $V=a[1+C(1/(\varphi+x))^2]^{\frac{1}{2}}$, of which C is a function of V_d, φ is a constant, and x is the distance of the source measured in units of "equivalent" diameter; the normal velocity of sound is a. The reduced times are defined by $\mathbf{K}a\delta t=[x^2+2\varphi x+(C+\varphi^2)]^{\frac{1}{2}}-1^{\varphi}$, where $\mathbf{K}=x/s$, s being the distance. The formulas are applicable to sources other than detonating charges.

65. II. The Propagation of a Shock-Wave in Air. L. THOMPSON AND N. RIFFOLT, *Naval Proving Ground, Dahlgren, Va.*—Charges of granular T.N.T. were detonated, obtaining times of transit of the pulse with oscillograph elements connected at six points to piezoelectric screens, as well as zero time by rupture of a wire around the charge. With the expression for reduced times given in Part I, these observations determine the constant φ of the general formula for velocity.

66. An Anomalous Conductivity in Powdered Tellurium. C. HAWLEY CARTWRIGHT, *University of Michigan.*—It was found that tellurium powder under 1000 kg/cm² pressure has ten times greater specific electrical conductivity than a single crystal of tellurium. The influence of the electrodes, particle size, surface condition and the medium surrounding the particles were investigated. Momentarily passing an electric current through the powder under pressure caused its volume to diminish; but, also the electrical conductivity decreased to that of the massive metal. A hydrostatic pressure of 20,000 kg/cm² increases the electrical conductivity of massive tellurium about a hundred times and the rate of increase rises with pressure. However, the suggestion that local pressures exist that are greatly in excess of the average pressure does not account entirely for the observed magnitude of the anomalous conductivity. It seems necessary to assume (1) the conductivity of tellurium increases even much more for nonuniform pressures than for hydrostatic pressures or (2) very sharp points occur between particles which produce strong enough electric fields to liberate conduction electrons. This anomalous effect is very small in absolute magnitude and would be masked in conducting metals.

67. Quasi-Crystalline Structure of Water from Infrared Data. C. HAWLEY CARTWRIGHT, *University of Michigan.*—Considerable evidence of a quasi-crystalline structure in liquids is obtained by considering the specific heat, dielectric constant, x-ray spectra, Raman spectra and viscosity. The transmission and reflection measurements of liquid water in the infrared from 15μ to 150μ show characteristics which are unlike the vapor spectra of free rotation and can be interpreted as due to a hindered rotation and hindered translation of the molecules as a whole in a quasi-crystalline structure. From the characteristic reflection and absorption frequencies (which are naturally broad), the intermolecular binding energies of the water molecules are calculated.

68. A High Dispersion Linear Scale Mass Spectrograph with Velocity and Direction Focusing. KENNETH T. BAINBRIDGE AND EDWARD B. JORDAN, National Research Fellow, *Harvard University.*—The simultaneous fulfillment of the conditions for high resolving power, high sensitivity and high dispersion has been obtained in an instrument for the investigation of isotopes. An important feature of the design is the linearity of the mass scale over a broad region of the recording plate. The ions diverging from the second slit S_2 are deflected through $\pi/\sqrt{2}$ radians by using the radial field method of electrostatic focusing introduced by Hughes and Rojansky which produces an energy spectrum in front of the third slit S_3. Beams of ions of all m/e values comprising up to 3 percent of this energy spectrum, traversing S_3, are further deflected through a mean angle of $\pi/3$ radians by a magnetic field which focuses the ions and produces a mass spectrum on the recording plate. The mean dispersion is 5 mm for one percent mass difference, the mean radius is 25.4 cm, and a resolving power of 12,000 has been attained. Over 140 mm on the plate the maximum divergences from linearity are $\pm 1/7000$. Sample spectra will be shown of $O-CH_4$, NH_2-OH, $H_2^1-D^2$, and other doublets.

69. The Cooling of a Surface Due to Photoelectric Emission. HUGHES M. ZENOR, *Rice Institute, Houston, Texas.*—The energy loss due to photoelectric emission from a platinum surface activated with a thin layer of caesium has been measured. Two similar photoelectric cells were illuminated with the same source of light. Each cell contained a small platinum plate coated with caesium, the temperature of which was measured with a Fe–Ni thermocouple. The thermocouple junctions of the two cells were connected in series with a galvanometer in such a manner that the e.m.f.'s of the two cells opposed each other. In this way it was possible to determine with the galvanometer the change in temperature when the photoelectric current was allowed to flow or was stopped in one of the cells. The photoelectric current was controlled by the potential of the anode. The value found for the energy loss per electron is consistent with present photoelectric theories.

70. The Sign of the Magnetic Moment of the Proton. I. I. RABI, J. M. B. KELLOGG, J. R. ZACHARIAS, *Columbia University.*—Because of the large value found for the magnetic moment of the proton, theory has not sufficed to predict the sign. We have made this measurement by a method of nonadiabatic transitions. A beam of neutral hydrogen atoms is first deflected in a weak inhomogeneous

magnetic field and then in a strong inhomogeneous field arranged to deflect in the opposite direction. The whole arrangement forms a "magnetic moment spectrograph." In a magnetically shielded region between the two fields is placed a set of wires carrying current, which produce weak, rapidly varying magnetic fields to induce non-adiabatic transitions between the different magnetic levels and a slit to sort out the different levels. On examining the moments from the two $m=0$ levels it was found that the state which has the negative over-all moment made the transitions. The state with positive over-all moment did not. The state with positive over-all moment is therefore the $F=0$ state. The h.f.s. multiplet is normal. The proton moment is therefore positive, i.e., it has the spin direction.

71. The Scattering of Electrons after Ionizing Collisions in Helium. MAX GOODRICH, *University of Minnesota.* An electron scattering apparatus has been used to determine, in helium, the angular distribution [for 100-volt incident electrons] of electrons scattered with various energy losses equal to or greater than the ionization loss. The curves show that, in general, the greater the energy loss the more uniform is the distribution in angle of the scattered electrons. There is a pronounced preference for forward scattering when the energy loss over that required for ionization is small. Integration over the entire solid angle gives the probability for scattering with different energy losses. Curves so obtained for this probability as a function of energy loss are in qualitative agreement with Wetzel's theoretical calculations except for electrons which have lost nearly all their energy or just slightly more than the ionization energy.

72. A Continuously Sensitive Cloud Chamber. ALEXANDER LANGSDORF, JR., *Massachusetts Institute of Technology.*—This is a preliminary note on the development of a continuously sensitive cloud chamber designed to overcome certain difficulties inherent in the familiar Wilson expansion chamber. Initial tests made with the new device have utilized the diffusion of methyl alcohol vapor vertically downward through an atmosphere of carbon dioxide gas. The alcohol vapor at 75 degrees centigrade and at atmospheric pressure is supplied to the top of the chamber from a steam heated boiler and diffuses vertically downward from the top to the bottom of the chamber, where a temperature of -45 degrees centigrade is maintained by a refrigerated plate. Under these conditions there is an intermediate region sufficiently supersaturated to show characteristic tracks due to cosmic-ray primaries and secondaries, as well as those due to secondary electrons from gamma-radiation. No difficulties due to turbulence have been encountered, and various types of tracks ranging in length from a fraction of an inch to nearly two feet have been observed in profusion. The number of cosmic-ray tracks visible in the course of a few minutes is comparable to the number photographed in several thousand exposures with the expansion method. Satisfactory photographs of the tracks have thus far not been obtained because of a continuous rain of condensed droplets which interferes with photography, though it does not obscure visibility of the tracks to the eye. Work is now in progress having for its object the elimination of the rain and the obtaining of satisfactory photographs.

73. Formula for Asymmetrical X-Ray Lines. ROY C. SPENCER, *University of Nebraska.*—Asymmetrical x-ray lines appear to have the form of $x=\pm(1/y-1)^{\frac{1}{2}}+sy$. The first term is merely the equation of the witch $y=1/(1+x^2)$ introduced by Hoyt. The second is the equation of the median line and has the effect of a shear of magnitude s along the x axis. The solution for y is a series, $y=1/(1+x^2)+sx/(1+x^2)^3-s/(1+x^2)^4$. The tangents at the half-maximum points intersect at the peak as does also the median line. *Criteria of Asymmetry.* (1) The author first used the ratio of the tangents at the half-maximum points. This is $R_1=-(2+s)/(2-s)$. (2) Allison uses the ratio of the half-widths at half-maximum. This is $R_2=-(2+s)/(2-s)$. (3) The shift in the peak, or the reciprocal slope s of the median line may be substituted in $R_3=-(2+s)/(2-s)$. Theoretically $R_1=R_2=R_3$, and all three criteria agree. *Application.* The equation is applied to an x-ray line ($s=0.2$) and to an absorption line in light ($s=0.8$) with good fit except at the extremities of the curve.

74. The Observation of Negative Hydrogen Ions. PAUL F. DARBY AND WILLARD H. BENNETT, *Ohio State University.*—Negative ions with mass approximately the same as that of the hydrogen atoms were drawn from a low voltage arc in water vapor. The ions passed through a hole in the anode of the discharge and were focused by a system of cylindrical electrodes on a willemite screen. The electrons were deflected out of the beam by a transverse magnetic field leaving the beam of negative ions but slightly displaced. A Faraday cage measured the current in the focused ion spot to be 1×10^{-8} ampere. Constricting the anode area, and using an oxide-coated filament as a cathode, was effective in raising the current to 8×10^{-8} ampere. The electron lens of cylindrical electrodes brings ions having all values of e/m to a focus at the same point. These beams will be used in a single section tube for nuclear disintegration work.

75. Debye-Waller Temperature Factor for Anisotropic Crystals. E. O. WOLLAN, *Washington University.*—In another paper to be presented at this meeting, C. Zener has extended to the case of anisotropic crystals the Debye-Waller theory for the effect of temperature on x-ray reflection. This extension predicts that the intensity of reflection will be a function of the angle which the reflecting plane makes with the principal axis. Recent measurements[1] of the structure factors for metallic zinc make it possible to compare the theory with experiment. The measured structure factors for two angles of reflection are considerably displaced from a smooth curve. However, when the anisotropic character of the zinc crystal is considered on the basis of the extended theory, it is found that the apparently anomalous reflecting power is in accord with the predictions of the theory.

[1] G. W. Brindley and F. W. Spiers, Phil. Mag. **20**, 865 (1935).

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